

FREE RESPONSE PROBLEM #1
Calculator Permitted

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20 \sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

FREE RESPONSE PROBLEM #2
Calculator NOT Permitted

| | | | | | |
|-------------------------------|---|-----|-----|------|-----|
| t (minutes) | 0 | 12 | 20 | 24 | 40 |
| $v(t)$ (meters per minute) | 0 | 200 | 240 | -220 | 150 |

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

FREE RESPONSE PROBLEM #3
Calculator Permitted

On a certain workday, the rate, measured in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$, the hours of operation. During the workday, the plant processes gravel at a rate of 100 tons per hour. When the unprocessed gravel arrives, it is emptied into 3 bins that will hold 200 tons of gravel each. If the three bins are full at any given time, delivery trucks must wait to unload. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel.

a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

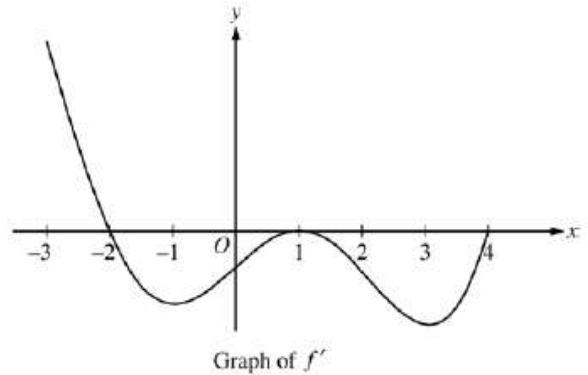
b. Find $\int_0^8 G(t) dt$. Using correct units, interpret your answer in the context of the problem.

c. Find an equation for $A(t)$, the amount of unprocessed gravel at the plant for any given time t . Using this equation, will the delivery truck arriving at $t = 7$ have to wait to unload its unprocessed gravel? Show your work and give a reason for your answer.

d. Write, but do not solve, an equation involving an integral that could be solved to determine the first time, t , when a truck would have to wait to unload its unprocessed gravel.

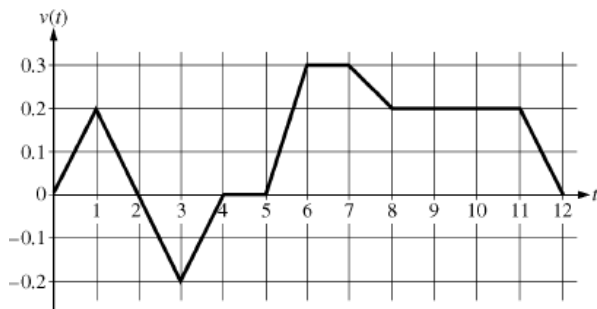
FREE RESPONSE PROBLEM #4
Calculator NOT Permitted

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

FREE RESPONSE PROBLEM #5
Calculator Permitted



Caren rides her bicycle along a straight road from home to school, leaving home at $t = 0$ minutes and arrived at school at $t = 12$ minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity, $v(t)$, in miles per minute is modeled by the piecewise-linear function whose graph is shown above.

a. Is Caren's speed increasing or decreasing at time $t = 7.5$ minutes? Completely explain your reasoning.

b. Find the value of $\int_0^{12} |v(t)| dt$. Explain, using appropriate units, what this value represents in the context of the problem.

c. At some point in time, Caren realizes that she left her calculus homework at home and turned around to go back home to get it. At what time t does she turn around? Give a reason for your answer.

d. Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is measured in miles per minute for $0 \leq t \leq 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

FREE RESPONSE PROBLEM #6
Calculator NOT Permitted

A particle is moving along a straight path. The velocity of the particle for $0 \leq t \leq 18$ is shown in the table below for selected values of t and velocity is a strictly increasing function.

| | | | | | | | |
|-----------------|----------|----------|-------------|-----------|-----------|-----------|-------------|
| t | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
| $v(t)$ m/sec | 0 | 7 | 10.5 | 12 | 13 | 14 | 14.5 |

- a. Using the midpoints of three subintervals of equal length, approximate the value of $\frac{1}{18} \int_0^{18} v(t) dt$. Using correct units, explain the meaning of the value of $\frac{1}{18} \int_0^{18} v(t) dt$.
- b. Find the average acceleration of the particle over the interval $6 \leq t \leq 18$. Express your answer using correct units.
- c. Find an approximation of $v'(6)$. Using correct units, explain what this value represents and state, providing justification, if the speed of the particle is increasing or decreasing at $t = 6$?
- d. Find $\int_3^9 v(t) dt$ using a left hand approximation of two subintervals of equal length. Then, use this value to find the position of the particle at $t = 3$ seconds if the position at $t = 9$ seconds is 60 meters, expression your answer using appropriate units.