AP Calculus
Unit 6 – Basic Integration & Applications

Day 7 Notes: Average Value of a Function

How have we found Average Velocity?

Avg Velocity $= \frac{p(a)-p(b)}{a-b}$

How have we found Average Acceleration?

Avg Acceleration $= \frac{v(a)-v(b)}{a-b}$

If $p(t)$, $v(t)$, and $a(t)$ represent position, velocity and acceleration defined for any time t, write an equivalent expression for each of the following integrals based on the fundamental theorem of calculus.

$\frac{1}{b-a}\int_a^b a(t)dt =$	$\frac{1}{b-a} \cdot v(b)-v(a)$	To what is this equivalent? Avg Acceler
$\frac{1}{b-a}\int_a^b v(t)dt =$	$\frac{1}{b-a} \cdot p(b)-p(a)$	To what is this equivalent? Avg Velocity

The average value of a function, $f(x)$, on an interval $[a, b]$ is defined to be:

$$\boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

Find the average value of the function $f(x) = x^3\sqrt{\sin^2 x}$ on the interval $1 \le x \le 3$. [Calculator]

math 9

$$\frac{1}{3-1}\int_1^3 x^3\sqrt{\sin^2 x} = \frac{1}{2}(11.696) = \boxed{5.848}$$

Find the average value of the function $f(x) = 2 - 4x$ on the interval $2 \le x \le 6$. [Noncalculator]

$$\frac{1}{6-2}\int_2^6 2-4x = \frac{1}{4}\left(\frac{2x}{1} - \frac{4x^2}{2} + C\Big]_2^6\right)$$

$$\frac{1}{4}\left[(2(6)-2(6)^2)-(2(2)-2(2)^2)\right]$$

$$\frac{1}{4}[12-72-4+8] = \frac{1}{4}[-56] = \boxed{-14}$$

A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t\sin^2\left(\frac{t}{14}\right)$. The rate at which the snow melts is modeled by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. Both $S(t)$ and $M(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 24$. At time $t = 0$, the slope holds 50 cubic yards of snow.

a. Compute the total volume of snow added to the mountain over the first 6-hour period.

math 9

$$\int_0^6 24 - t\sin^2\left(\frac{t}{14}\right)dt = \boxed{142.413 \text{ yd}^3}$$

b. Find the value of $\int_0^6 M(t)dt$ and $\frac{1}{6}\int_0^6 M(t)dt$. Using correct units of measure, explain what each represents in the context of this problem.

math 9

$$\int_0^6 10 + 8\cos\left(\frac{t}{3}\right)dt = \boxed{81.823 \text{ yd}^3}$$ → TOTal volume of snow that has melted in first 6 hrs

$$\frac{1}{6}\int_0^6 10 + 8\cos\left(\frac{t}{3}\right)dt = \boxed{13.637 \text{ yd}^3/\text{hr}}$$ → Avg value of the rate at which the snow is melting in first 6hr.

c. Is the volume of snow increasing or decreasing at time $t = 4$? Justify your answer.

$$A(t) = 50 + \int_0^t S(t)dt - \int_0^t m(t)dt$$

$$A'(t) = S(t) - m(t)$$

$$A'(4) = S(4) - m(4) = 23.682 - 11.882 = \boxed{11.800 \text{ yd}^3/\text{hr}}$$

Since $A'(4) > 0$, the volume is incr.

d. How much snow is on the slope after 5 hours? Show your work.

math 9

$$A(5) = 50 + \int_0^5 S(t)dt - \int_0^5 M(t)dt$$

$$= 50 + 119.225 - 73.890 = \boxed{95.335 \text{ yd}^3}$$

e. Suppose the snow machine is turned off at time $t = 10$. Write, but do not solve, an equation that could be solved to find the time $t = K$ when the snow would all be melted.

math 9

$$A(10) = 50 + \int_0^{10} S(t)dt - \int_0^{10} M(t)dt = 50 + 228.619 - 95.426 = \boxed{183.193}$$

$$\boxed{183.193 - \int_{10}^{K} 10 + 8\cos\left(\frac{t}{3}\right)dt = 0}$$

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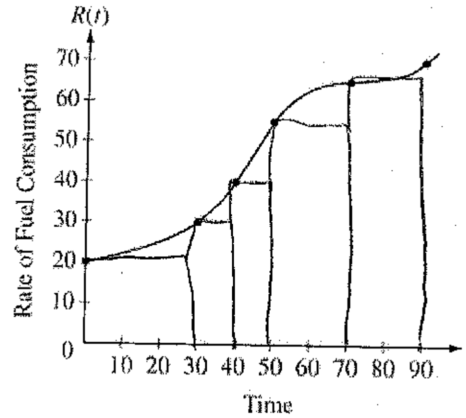
Problem #3



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
max

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \dfrac{R(40) - R(50)}{40 - 50} \approx \dfrac{40 - 55}{-10} = \boxed{1.5 \text{ gal/min}^2}$

(b) $\boxed{R''(45) = 0}$ since $R'(t)$ has a maximum at $t = 45$.

(c) $\int_0^{90} R(t)\,dt = (30)(20) + (10)(30) + (10)(40) + 20(55) + 20(65) = \boxed{3700}$

$\boxed{\text{Less b/c } R(t) \text{ is incr on } 0 \le t \le 90}$

(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed in the first b minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gal/min during the first b minutes.