AP Calculus AB
Unit 6 – Day 7 – Assignment

Name: Answer Key*

1. Using a right Riemann sum over the given intervals, estimate $\int_5^{35} F(t)dt$

t	5	13	22	27	35
$F(t)$	44	12	13	17	22

A. 730
B. 661
C. 564
D. 474
E. 325

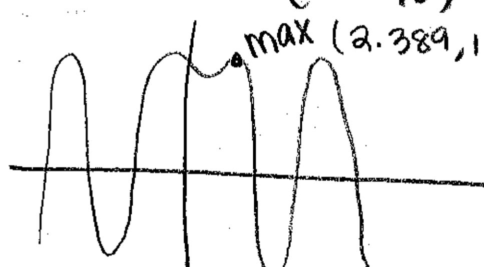
$(8)(12) + (9)(13) + (5)(17) + (8)(22)$
$= 474$

2. For the first six seconds of driving, a car accelerates at a rate of $a(t) = 10\sin\left(1+\frac{t^2}{10}\right)$ meters per second^2. Which one of the following expressions represents the velocity of the car when it first begins to decelerate?

← Acceleration begins to decrease

A. $\int_0^{0.775} a(t)dt$
B. $\int_0^{2.389} a(t)dt$
C. $\int_0^{1.715} a(t)dt$
D. $\int_0^{4.627} a(t)dt$
E. $\int_0^{3.830} a(t)dt$

$10\sin\left(1+\frac{t^2}{10}\right)$ ← Graph on Calc



Begins to decr. first at $t = 2.389$

3. The rate at which gas is flowing through a large pipeline is given in thousands of gallons per month in the chart below.

t (months)	0	3	6	9	12
$R(t)$ (1000 gallons per month)	43	62	56	60	68

Use a midpoint Riemann sum with two equal subintervals to approximate the number of gallons that pass through the pipeline in a year.

A. 594,000
B. 672,000
C. 732,000
D. 744,000
E. 1,068,000

$6(62) + 6(60) = 732$

4. Let f be a continuous function on the closed interval [1, 11]. If the values of f are given below at three points, use a trapezoidal approximation to find $\int_1^{11} f(x)dx$ using two subintervals.

A. 165
B. 172
~~C. 190.5~~
D. 40
E. 80

x	1	9	11
$f(x)$	23	14	10

$\frac{1}{2}(8)(23+14)+\frac{1}{2}(2)(14+10)$

$148 + 24 = 172$

5. If $\int_a^b f(x)dx = 2a - 3b$, then $\int_a^b [f(x)+3]dx =$

A. $2a - 3b + 3$
~~B. $3b - 3a$~~
C. $-a$
D. $5a - 6b$
E. $a - 6b$

$\underbrace{\int_a^b f(x)\,dx}_{} + \int_a^b 3\,dx$

$2a - 3b + 3x\Big]_a^b$

$2a - \cancel{3b} + \cancel{3(b)} - 3(a) = -a$

Use the table below to answer questions 6 and 7. Suppose the function $f(x)$ is a continuous function and f is the derivative of $F(x)$.

x	0	1	2	3
$f(x)$	−1	0	1	−2
$F(x)$	4	3	A	8

6. What is $\int_1^3 f(x)dx$? $= F(3) - F(1) = 8 - 3 = 5$

A. 5
B. 8
C. 4
D. 19
E. Cannot be determined

7. If the area under the curve of $f(x)$ on the interval $0 \le x \le 2$ is equal to the area under the curve $f(x)$ on the interval $2 \le x \le 3$, then what is the value of A?

A. 4
B. 2
~~C. 5.5~~
D. 6
E. Cannot be determined

$\int_0^2 f(x)dx = \int_2^3 f(x)dx$

$F(2) - F(0) = F(3) - F(2)$

$A - 4 = 8 - A$

$2A = 12$

$A = 6$

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Problem #1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30, \quad \text{rate}$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = \int_0^{30} 82 + 4\sin\left(\frac{t}{2}\right) dt \overset{\text{math9}}{=} 2474.078 = \boxed{2474 \text{ cars}}$

(b) $F'(7) = -1.873$ math8

$\boxed{\text{Since } F'(7) < 0 \text{, the traffic flow is decreasing at } t = 7.}$

(c) $\frac{1}{15-10}\int_{10}^{15} 82 + 4\sin\left(\frac{t}{2}\right) dt = \frac{1}{5}(409.496) = \boxed{81.899 \text{ cars/min}}$

(d) $\frac{F(10) - F(15)}{10-15} = \frac{78.164 - 85.752}{-5} = \boxed{1.518 \text{ cars/min}^2}$

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Problem #2

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

(a) $W'(12) \approx \dfrac{W(9)-W(15)}{9-15} \approx \dfrac{24-21}{-6} = \boxed{-0.5\ °C/\text{day}}$

(b) $\frac{1}{2}(3)(20+31) + \frac{1}{2}(3)(31+28) + \frac{1}{2}(3)(28+24) + \frac{1}{2}(3)(24+22) + \frac{1}{2}(3)(22+21)$

$76.5 + 88.5 + 78 + 69 + 64.5 = 376.5 = \int_0^{15} W(t)$

$\frac{1}{15-0}\int_0^{15} W(t) = \frac{1}{15}(376.5) = \boxed{25.1°C}$

(c) $P'(t) = (10)(e^{-t/3}) + (10t)(e^{-t/3})(-\frac{1}{3}) = 10e^{-t/3} - \frac{10}{3}te^{-t/3}$

$P'(12) = 10e^{-12/3} - \frac{10}{3}(12)e^{-12/3} = 10e^{-4} - 40e^{-4} = -30e^{-4} = \boxed{-0.549\ °C/\text{day}}$

This means that the temperature is decreasing at the rate of 0.549 °C/day when t=12 days.

(d) $\frac{1}{15}\int_0^{15} 20 + 10te^{-t/3}\,dt = \frac{1}{15}(386.362) = \boxed{25.757°C}$

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