SOLVING TRIG EQUATIONS

1. Use identities to make sure all angles are the **same**.
2. If all angles are the same but they are multiple angles, **adjust the domain** to reflect the multiple. (For example, if all the angles are $2x$, and you're solving on the interval $0 \le x < 2\pi$, adjust the interval to $0 \le 2x < 4\pi$, which means you want all appropriate angles in 2 full revolutions.
3. Use identities to minimize the number of trig functions, if possible.
4. Rearrange the equation so one side = 0, then **factor**.
5. In a few cases, you might need to square both sides.
6. Check, especially if there were denominators or you had to square both sides.

Solve each equation on the interval [0, 2π). Show your work on separate paper.

1) $2\cos^2 x - 5\cos x = -2$

2) $2\tan x \sin x + 2\sin x = \tan x + 1$
(Hint: factor by grouping)

3) $\sin^2 x + \cos 2x - \cos x = 0$

4) $4\cos^2 2x - 3 = 0$

5) $\sqrt{3}\tan 3x + 1 = 0$

6) $3\tan^2 x + 4\sec x = -4$

7) $\cos 2x \cos x - \sin 2x \sin x = 1$
(Hint: simplify with angle-sum identity)

8) $\cos 2x + 3\cos x - 1 = 0$

9) $\sqrt{3}\cot x \sin x + 2\cos^2 x = 0$

10) $\sin^2 2x - 3\sin 2x + 2 = 0$

11) $\tan^2 3x = \sqrt{3}\tan 3x$

12) $4\tan x + \sin 2x = 0$

13) $\sin 4x = \cos 2x$

14) $3\tan^2 2x + 4\sec 2x = -4$

Solve for ALL values of *x*.

15) $\cos x \tan x - \sin^2 x = 0$

16) $3\cos 2x - 5\cos x = 1$

17) $\cos x = 3\cos x - 2$

18) $\dfrac{\tan x - \sin x}{\tan x + \sin x} = \dfrac{\sec x - 1}{\sec x + 1}$

19) $\sin x + 1 = \cos x$ (Hint: square both sides)

20) $\cos 2x \cos 3x + \sin 2x \sin 3x = \dfrac{1}{2}$
(Hint: simplify with angle-sum identity)

ANSWERS

1. $\frac{\pi}{3}, \frac{5\pi}{3}$

2. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{4}, \frac{7\pi}{4}$

3. $0, \frac{\pi}{2}, \frac{3\pi}{2}$

4. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

5. $\frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}, \frac{23\pi}{18}, \frac{29\pi}{18}, \frac{35\pi}{18}$

6. π

7. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

8. $\frac{\pi}{3}, \frac{5\pi}{3}$

9. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$

10. $\frac{\pi}{4}, \frac{5\pi}{4}$

11. $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3},$
    $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$

12. $0, \pi$

13. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

14. $\frac{\pi}{2}, \frac{3\pi}{2}$

15. $\pi n, \frac{\pi}{2}+2\pi n$

16. $\frac{2\pi}{3}+2\pi n, \frac{4\pi}{3}+2\pi n$

17. $2\pi n$

18. All x, $x \neq (2n+1)\frac{\pi}{2}$

19. $\frac{3\pi}{2}+2\pi n, 2\pi n$

20. $\frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n$

Solving Trig Equations **

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1 FACTOR!

$2\cos^2 x - 5\cos x = -2$ $[0, 2\pi)$

$2\cos^2 x - \underline{5\cos x} + 2 = 0$

$\dfrac{4}{-4 \mid -1}$

$(2\cos^2 x - 4\cos x)(-1\cos x + 2) = 0$

$2\cos x(\cos x - 2) - 1(\cos x - 2)$

$(2\cos x - 1)(\cos x - 2)$

$2\cos x = 1$ $\quad$ $\cancel{\cos x = 2}$

$\cos x = \frac{1}{2}$

$x = \boxed{\frac{\pi}{3}}\ \boxed{\frac{5\pi}{3}}$

* 2 $[0, 2\pi)$

Factor by grouping!

$2\tan x \sin x + 2\sin x = \tan x + 1$

$(2\tan x \sin x + 2\sin x)(-\tan x - 1) = 0$

$2\sin x(\tan x + 1) - 1(\tan x + 1)$

$(2\sin x - 1)(\tan x + 1) = 0$

$2\sin x = 1$ $\quad$ $\tan x = -1$

$\sin x = 1/2$

$x = \boxed{\frac{\pi}{6}}\ \boxed{\frac{5\pi}{6}}\ \boxed{\frac{3\pi}{4}}\ \boxed{\frac{7\pi}{4}}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$[0, 2\pi)$ (3) $\underbrace{\sin^2 x} + \underbrace{\cos 2x} - \cos x = 0$

Change everything to cos x!

Then Factor

$\cancel{1} - \cos^2 x + 2\cos^2 x - \cancel{1} - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0 \qquad \cos x = 1$

$x = \boxed{\frac{\pi}{2}} \quad \boxed{\frac{3\pi}{2}} \quad \boxed{0}$

(4) $4\cos^2(2x) - 3 = 0$ new interval: $[0, 4\pi)$

$4\cos^2(2x) = 3$

$\cos^2(2x) = \frac{3}{4}$

$\cos(2x) = \pm \frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{6} \qquad 2x = \frac{5\pi}{6} \qquad 2x = \frac{7\pi}{6} \qquad 2x = \frac{11\pi}{6}$

$12x = \pi \qquad 12x = 5\pi \qquad 12x = 7\pi \qquad 12x = 11\pi$

$\boxed{x = \frac{\pi}{12}} \qquad \boxed{x = \frac{5\pi}{12}} \qquad \boxed{x = \frac{7\pi}{12}} \qquad \boxed{x = \frac{11\pi}{12}}$

add $\frac{2\pi}{2}$ to each answer

$\frac{\pi}{12} + \pi \qquad \frac{5\pi}{12} + \pi \qquad \frac{7\pi}{12} + \pi \qquad \frac{11\pi}{12} + \pi$

$\frac{\pi}{12} + \frac{12\pi}{12} \qquad \frac{5\pi}{12} + \frac{12\pi}{12} \qquad \frac{7\pi}{12} + \frac{12\pi}{12} \qquad \frac{11\pi}{12} + \frac{12\pi}{12}$

$\boxed{x = \frac{13\pi}{12}} \qquad \boxed{x = \frac{17\pi}{12}} \qquad \boxed{x = \frac{19\pi}{12}} \qquad \boxed{x = \frac{23\pi}{12}}$

$\ast$ (5) $\sqrt{3}\tan 3x + 1 = 0$ new interval: $[0, 6\pi)$

$\sqrt{3}\tan 3x = -1$

$\tan 3x = \frac{-1}{\sqrt{3}}$ $\Rightarrow$

$\frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$3x = \frac{5\pi}{6}$ $3x = \frac{11\pi}{6}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

one $-$

$18x = 5\pi$ $18x = 11\pi$

$\boxed{x = \frac{5\pi}{18}}$ $\boxed{x = \frac{11\pi}{18}}$

add $\frac{2\pi}{3}$ to each answer (two times)

$\frac{5\pi}{18} + \frac{2\pi}{3}$ $\frac{11\pi}{18} + \frac{2\pi}{3}$

$\frac{5\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{17\pi}{18}}$ $\frac{11\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{23\pi}{18}}$

$\frac{17\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{29\pi}{18}}$ $\frac{23\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{35\pi}{18}}$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$[0, 2\pi)$ ⑥ $3\tan^2 x + 4\sec x = -4$

Change to all sec x & Factor

$3(\sec^2 x - 1) + 4\sec x = -4$

$3\sec^2 x - 3 + 4\sec x = -4$

$3\sec^2 x + \underline{4\sec x} + 1 = 0$

$\frac{3}{3 \mid 1}$

$(3\sec^2 x + 3\sec x) + (1\sec x + 1) = 0$

$3\sec x(\sec x + 1) + 1(\sec x + 1)$

$(3\sec x + 1)(\sec x + 1) = 0$

$\sec x = -\frac{1}{3}$ $\quad$ $\sec x = -1$

$\downarrow$ $\quad$ $\downarrow$

$\cancel{\cos x = -3}$ $\quad$ $\cos x = -1$

$\boxed{\pi}$

⑦ $\underbrace{\cos 2x \cos x - \sin 2x \sin x}$ $= 1$

Use angle-sum identity for cosine

$\cos(2x + x) = 1$

$\cos(3x) = 1$ $\quad$ new interval $[0, 6\pi)$

$\downarrow$

$3x = 0$

$\boxed{x = 0}$

add $\frac{2\pi}{3}$ two times!

$0 + \frac{2\pi}{3} = \boxed{\frac{2\pi}{3}}$

$\frac{2\pi}{3} + \frac{2\pi}{3} = \boxed{\frac{4\pi}{3}}$

8) $\cos 2x + 3\cos x - 1 = 0$

Change $\cos 2x$ to $\cos^2 x - 1$

FACTOR!

$2\cos^2 x - 1 + 3\cos x - 1 = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

$\frac{-4}{4 \mid -1}$

$(2\cos^2 x + 4\cos x)(-1\cos x - 2) = 0$

$2\cos x(\cos x + 2) - 1(\cos x + 2) = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cancel{\cos x = -2}$

$\boxed{x = \frac{\pi}{3}}$ $\boxed{x = \frac{5\pi}{3}}$

9) $\sqrt{3}\cot x \sin x + 2\cos^2 x = 0$

Change $\cot x = \frac{\cos x}{\sin x}$

$\sqrt{3}\,\frac{\cos x}{\cancel{\sin x}}\,\cancel{\sin x}$

$\sqrt{3}\cos x + 2\cos^2 x = 0$

$2\cos^2 x + \sqrt{3}\cos x = 0$ $\quad$ Factor!

$\cos x(2\cos x + \sqrt{3}) = 0$

$\cos x = 0$ $\quad$ $\cos x = -\frac{\sqrt{3}}{2}$

$\boxed{\frac{\pi}{2}}$ $\boxed{\frac{3\pi}{2}}$ $\boxed{\frac{5\pi}{6}}$ $\boxed{\frac{7\pi}{6}}$

$\cancel{X}$ (10) $\sin^2 2x - 3\sin 2x + 2 = 0$

$(\sin 2x - 2)(\sin 2x - 1) = 0$ $\frac{2}{-2 | -1}$ Factor!

$\cancel{\sin(2x) = 2}$ $\sin(2x) = 1$

new interval $[0, 4\pi)$

$2x = \frac{\pi}{2}$

$4x = \pi$

$\boxed{x = \frac{\pi}{4}}$

add $\frac{2\pi}{2}$ one time! $\frac{\pi}{4} + \pi = \frac{\pi}{4} + \frac{4\pi}{4} = \boxed{\frac{5\pi}{4}}$

(11) $\tan^2 3x = \sqrt{3}\tan 3x$

$\tan^2 3x - \sqrt{3}\tan 3x = 0$

$\tan 3x(\tan 3x - \sqrt{3}) = 0$

new interval $[0, 6\pi)$

$\tan(3x) = 0$ $\tan(3x) = \sqrt{3}$

$\frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\frac{y}{x} = \frac{0}{1}$

$(\pm 1, 0)$

add $2\pi/3$, two times!

$3x = 0$ $\boxed{x = 0}$ $\boxed{2\pi/3}$ $\boxed{4\pi/3}$

$3x = \pi$ $\boxed{x = \frac{\pi}{3}}$ $\boxed{\pi}$ $\boxed{5\pi/3}$

$3x = \frac{\pi}{3}$ $9x = \pi$ $\boxed{x = \pi/9}$ $\boxed{7\pi/9}$ $\boxed{13\pi/9}$

$3x = \frac{4\pi}{3}$ $9x = 4\pi$ $\boxed{x = 4\pi/9}$ $\boxed{\frac{10\pi}{9}}$ $\boxed{\frac{16\pi}{9}}$

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12 $4\tan x + \sin 2x = 0$

$[0, 2\pi)$

$4\left(\frac{\sin x}{\cos x}\right) + 2\sin x\cos x = 0$

$\frac{4\sin x}{\cos x} + \frac{2\sin x\cos x(\cos x)}{(\cos x)} = 0$

$\frac{4\sin x + 2\sin x\cos^2 x}{\cos x} = 0$

$4\sin x + 2\sin x\cos^2 x = 0$

$\sin^2 x + \cos^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$

$4\sin x + 2\sin x(1 - \sin^2 x) = 0$
$4\sin x + 2\sin x - 2\sin^3 x = 0$

$-2\sin^3 x + 6\sin x = 0$

$[0, 2\pi)$

$-2\sin x(\sin^2 x - 3) = 0$

$-2\sin x = 0 \qquad \sin^2 x = 3$

$\sin x = 0 \qquad \cancel{\sin x = \sqrt{3}}$

$\boxed{0, \pi}$

(13) $\sin 4x = \cos 2x$

$\sin(2x+2x)$

$\underline{\sin 2x \cos 2x} + \underline{\cos 2x \sin 2x} = \cos 2x$

$2\sin 2x \cos 2x - \cos 2x = 0$

$\cos 2x(2\sin 2x - 1) = 0$

new interval $[0, 4\pi)$

$\cos 2x = 0$ $\qquad$ $\sin 2x = \frac{1}{2}$

$2x = \frac{\pi}{2}$ $\qquad$ $2x = \frac{3\pi}{2}$ $\qquad$ $2x = \frac{\pi}{6}$ $\qquad$ $2x = \frac{5\pi}{6}$

add $\frac{2\pi}{2}$ to each

$4x = \pi$ $\qquad$ $4x = 3\pi$ $\qquad$ $12x = \pi$ $\qquad$ $12x = 5\pi$

$\boxed{x = \frac{\pi}{4}}$ $\qquad$ $\boxed{x = \frac{3\pi}{4}}$ $\qquad$ $\boxed{x = \frac{\pi}{12}}$ $\qquad$ $x = \boxed{\frac{5\pi}{12}}$

$\frac{\pi}{4} + \pi$ $\qquad$ $\frac{3\pi}{4} + \frac{4\pi}{4} = \boxed{\frac{7\pi}{4}}$ $\qquad$ $\frac{\pi}{12} + \frac{12\pi}{12}$ $\qquad$ $\frac{5\pi}{12} + \frac{12\pi}{12}$

$\frac{\pi}{4} + \frac{4\pi}{4} = \boxed{\frac{5\pi}{4}}$ $\qquad$ $\boxed{\frac{3\pi}{12}}$ $\qquad$ $\boxed{\frac{17\pi}{12}}$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

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(14) $3\tan^2 2x + 4\sec 2x = -4$

Change $\tan^2(2x)$ to $\sec^2(2x) - 1$

$3(\sec^2 2x - 1) + 4\sec 2x = -4$

$3\sec^2 2x - 3 + 4\sec 2x = -4$

FACTOR!

$3\sec^2(2x) + \underline{4\sec(2x)} + 1 = 0$

$\frac{3}{3 \mid 1}$

$(3\sec^2(2x) + 3\sec(2x)) + (\sec(2x) + 1)$

new interval $[0, 4\pi)$

$3\sec(2x)(\sec 2x + 1) + 1(\sec 2x + 1)$

$(3\sec 2x + 1)(\sec 2x + 1) = 0$

$3\sec 2x = -1$

$\sec 2x = \frac{-1}{3}$

$\cancel{\cos(2x) = -3}$

$\sec 2x = -1$

$\cos 2x = -1$

$2x = \pi$

$\boxed{x = \frac{\pi}{2}}$

$\frac{\pi}{2} + \pi = \boxed{\frac{3\pi}{2}}$

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(15) $\cos x \tan x - \sin^2 x = 0$ all

$\cancel{\cos x}\left(\frac{\sin x}{\cancel{\cos x}}\right) - \sin^2 x = 0$

$\sin x - \sin^2 x = 0$

$\sin x(1 - \sin x) = 0$

$\downarrow$ $\downarrow$

$\sin x = 0$ $1 - \sin x = 0$

$-\sin x = -1$

$\sin x = 1$

$0, \pi, 2\pi$ $\frac{\pi}{2}$

$\boxed{\pi n \text{ \& } \frac{\pi}{2} + 2\pi n;\ n = 0, \pm 1, \pm 2, \ldots}$

(16) $3\cos 2x - 5\cos x = 1$ all

Replace $\cos 2x$ w/ $2\cos^2 x - 1$

$3(2\cos^2 x - 1) - 5\cos x = 1$

$6\cos^2 x - 3 - 5\cos x - 1 = 0$

$6\cos^2 x - 5\cos x - 4 = 0$

$(6\cos^2 x - 8\cos x) + 3\cos x - 4$

$2\cos x(3\cos x - 4) + 1(3\cos x - 4)$

-24

$-8 \mid 3$

$(2\cos x + 1)(3\cos x - 4) = 0$

$\cos x = -\frac{1}{2}$ $\quad \cancel{\cos x = \frac{4}{3}}$

$\downarrow$

$\frac{2\pi}{3}, \quad \frac{4\pi}{3}$

$$\boxed{\frac{2\pi}{3} + 2\pi n, \quad \frac{4\pi}{3} + 2\pi n \ ; \quad n = 0, \pm 1, \pm 2, \ldots}$$

(17) $\cos x = 3\cos x - 2$

$2 = 2\cos x$

$1 = \cos x$

$\downarrow$

$0, 2\pi$

$\boxed{2\pi n,\ n = 0, \pm 1, \pm 2, \ldots}$

subtract over $\cos x$

(18) $\dfrac{\tan x - \sin x}{\tan x + \sin x} = \dfrac{\sec x - 1}{\sec x + 1}$

$\dfrac{\sin x}{\cos x}$ $\quad$ $\dfrac{1}{\cos x}$

$(\tan x - \sin x)(\sec x + 1) = (\tan x + \sin x)(\sec x - 1)$

$\cancel{\tan x \sec x} + \tan x - \sin x \sec x - \cancel{\sin x} = \cancel{\tan x \sec x} - \tan x + \sin x \sec x - \cancel{\sin x}$

$\tan x - \sin x \sec x = -\tan x + \sin x \sec x$

$2\tan x - 2\sin x \sec x = 0$

$2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\left(\dfrac{1}{\cos x}\right) = 0$

$2\tan x - 2\tan x = 0$

$0 = 0$ ✓

$\cos x \neq 0$

$\downarrow$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{\text{all } x,\ x \neq (2n+1)\frac{\pi}{2}}$

$\downarrow$

This means <u>odd</u> intervals of $\frac{\pi}{2}$

(19) $(\sin x+1)^2=(\cos x)^2$

$(\sin x+1)(\sin x+1)=\cos^2 x$

$\sin^2 x+2\sin x+1=\cos^2 x$ $\qquad$ $\sin^2 x+\cos^2 x=1$

$\sin^2 x+2\sin x+\cancel{1}=\cancel{1}-\sin^2 x$

$-2\sin^2 x+2\sin x=0$

$-2\sin x(\sin x-1)=0$

$-2\sin x=0$ $\qquad$ $\sin x=1$

$\sin x=0$ $\qquad$ $\downarrow$

$\downarrow$

$0,\ \pi,\ 2\pi$ $\qquad$ $\frac{\pi}{2},\ \frac{3\pi}{2}$

Since we squared both sides, check for extraneous solutions!

$\cancel{\sin\pi+1=\cos\pi}$ $\qquad$ $\cancel{\sin\frac{\pi}{2}+1=\cos\frac{\pi}{2}}$

$\cancel{0+1=-1}$ $\qquad$ $\cancel{1+1=0}$

$\sin 2\pi+1=\cos 2\pi$ $\qquad$ $\sin\frac{3\pi}{2}+1=\cos\frac{3\pi}{2}$

$0+1=1$ ✓ $\qquad$ $-1+1=0$ ✓

$$\boxed{2\pi n,\ \frac{3\pi}{2}+2\pi n;\ n=0,\pm1,\pm2,\dots}$$

(20) $\boxed{\cos 2x \cos 3x + \sin 2x \sin 3x} = \frac{1}{2}$

$\cos(2x + 3x) = \frac{1}{2}$

$\cos(5x) = \frac{1}{2}$

$\downarrow$

$5x = \frac{\pi}{3}$ $\qquad$ $5x = \frac{5\pi}{3}$

$15x = \pi$

$x = \frac{\pi}{15}$ $\qquad$ $15x = 5\pi$

$x = \frac{5\pi}{15} = \frac{\pi}{3}$

add $2\pi/5$ b/c it is $5x$

$5\left(\frac{\pi}{15} + \frac{2\pi}{5}n\right), \left(\frac{\pi}{3} + \frac{2\pi}{5}n\right)5$

$\boxed{\frac{\pi}{3} + 2\pi n}$, $\boxed{\frac{5\pi}{3} + 2\pi n}$ $\quad$ $n = 0, \pm 1, \pm 2, \ldots$