AP Calculus
Unit 6 – Basic Integration & Applications

Day 6 Notes: Applications of the Derivative and the Definite Integral

$$\frac{d}{dx}[\text{AMOUNT}] = \text{The rate at which that amount is changing}$$

For example, if water is being drained from a swimming pool and $R(t)$ represents the amount of water, measured in cubic feet, that is in a swimming pool at any given time, measured in hours, then $R'(t)$ would represent the rate at which the amount of water is changing.

$$\frac{d}{dx}[R(t)] = R'(t)$$

What would the units of $R'(t)$ be?____________________________

$$\int_a^b \text{RATE} = \text{AMOUNT OF CHANGE}$$

In the context of the example situation above, explain what this value represents:

$\int_a^b R'(t)dt = R(b) - R(a)$.

The table given below represents the velocity of a particle at given values of t, where t is measure in minutes.

t minutes	0	5	10	15	20	25	30
$v(t)$ ft/minute	0	1.6	2.7	3.1	2.4	1.6	0

a. Approximate the value of $\int_0^{30} v(t)dt$ using a midpoint Riemann Sum. Using correct units of measure, explain what this value represents.

b. What is the value of $\int_{5}^{25} a(t)dt$, and using correct units, explain what this value represents.

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes.

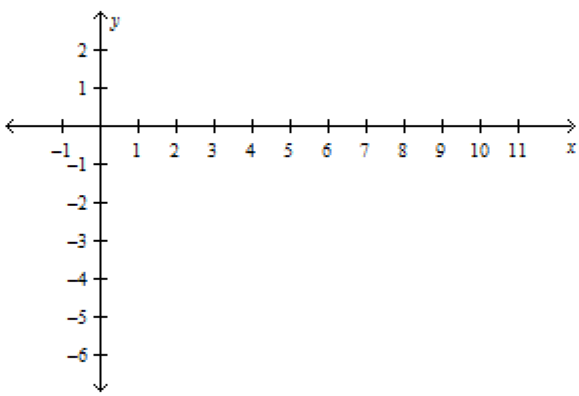
t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

Using the data in the table, estimate the value of $W'(12)$. Using correct units, interpret the meaning of this value in the context of this problem.

Use the data in the table to evaluate $\int_{0}^{20} W'(t)dt$. Using correct units, interpret the meaning of this integral in the context of this problem.

For $20 \le t \le 25$, the function W that models the water temperature has a first derivative given by the function $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on this model, what is the temperature of the water at time
$t = 25$?

A pan of biscuits is removed from an oven at which point in time, t = 0 minutes, the temperature of the biscuits is 100°C. The rate at which the temperature of the biscuits is changing is modeled by the function $B'(t) = -13.84e^{-0.173t}$.

Find the value of $B'(3)$. Using correct units, explain the meaning of this value in the context of the problem.
Sketch the graph of $B'(t)$ on the axes below. Explain in the context of the problem why the graph makes sense. 
At time t = 10, what is the temperature of the biscuits? Show your work.

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. During the first 5 days of a 60-day period, 3 millimeters of rainfall had been collected. The height of water in the can is modeled by the function, S, where $S(t)$ is measured in millimeters and t is measured in days for $5 \le t \le 60$. The rate at which the height of the water is rising is given by the function $S'(t) = 2\sin(0.03t) + 1.5$.

Find the value of $\int_{10}^{15} S'(t)dt$. Using correct units, explain the meaning of this value in the context of this problem.

At the end of the 60-day period, what is the volume of water that had accumulated in the can? Show your work.

The rate at which people enter an auditorium for a concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. V.I.P. tickets were sold to 100 people who are already in the auditorium when the when the doors open at $t = 0$ for general admission ticket holders to enter. The doors close and the concert begins at $t = 2$.

If all of the V.I.P. ticket holders stayed for the start of the concert, how many people are in the auditorium when the concert begins?

AP Calculus AB **Name: ______________________________**
Unit 6 – Day 6 – Assignment

At time $t = 0$, there are 120 pounds of sand in a conical tank. Sand is being added to the tank at the rate of $S(t) = 2e^{\sin^2 t} + 2$ pounds per hour. Sand from the tank is used at a rate of $R(t) = 5\sin^2 t + 3\sqrt{t}$ pounds per hour. The tank can hold a maximum of 200 pounds of sand.

1. Find the value of $\int_0^4 S(t)\,dt$. Using correct units, what does this value represent?

2. Find the value of $\int_1^3 R(t)\,dt$. Using correct units, what does this value represent?

3. Find the value of $\frac{1}{4}\int_0^4 S(t)\,dt$. Using correct units, what does this value represent?

4. Write a function, $A(t)$, containing an integral expression that represents the amount of sand in the tank at any given time, t.

5. How many pounds of sand are in the tank at time $t = 7$?

6. After time $t = 7$, sand is not used any more. Sand is, however, added until the tank is full. If k represents the value of t at which the tank is at maximum capacity, write, but do not solve, an equation using an integral expression to find how many hours it will take before the tank is completely full of sand.

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Problem #2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.