AP Calculus AB
Unit 6 – Day 6 – Assignment

Name: Answer Key*

At time $t = 0$, there are 120 pounds of sand in a conical tank. Sand is being added to the tank at the rate of $S(t) = 2e^{\sin^2 t} + 2$ pounds per hour. Sand from the tank is used at a rate of $R(t) = 5\sin^2 t + 3\sqrt{t}$ pounds per hour. The tank can hold a maximum of 200 pounds of sand.

1. Find the value of $\int_0^4 S(t)dt$. Using correct units, what does this value represent?

math 9

$\int_0^4 2e^{\sin^2 t} + 2\,dt = \boxed{21.173 \text{ pounds}}$ of sand were added to the tank in the first 4 hours.

2. Find the value of $\int_1^3 R(t)dt$. Using correct units, what does this value represent?

math 9

$\int_1^3 5\sin^2 t + 3\sqrt{t}\,dt = \boxed{14.878 \text{ pounds}}$ of sand was used b/t 1 hour & 3 hours.

3. Find the value of $\frac{1}{4}\int_0^4 S(t)dt$. Using correct units, what does this value represent?

math 9

$\frac{1}{4}\int_0^4 2e^{\sin^2 t} + 2\,dt = \boxed{5.293 \text{ pounds per hour}}$

represents the avg # of pounds per sand added to the tank per hour from 1 to 4 hours.

4. Write a function $A(t)$ containing an integral expression that represents the amount of sand in the tank at any given time, t.

$$\boxed{A(t) = 120 + \int_0^t S(t)\,dt - \int_0^t R(t)\,dt}$$

5. How many pounds of sand are in the tank at time $t = 7$?

$$A(7) = 120 + \underbrace{\int_0^7 S(t)\,dt}_{\text{math 9}} - \underbrace{\int_0^7 R(t)\,dt}_{\text{math 9}}$$

$$= 120 + 37.722 - 53.302 = \boxed{104.420 \text{ pounds}}$$

6. After time $t = 7$, sand is not used any more. Sand is, however, added until the tank is full. If k represents the value of t at which the tank is at maximum capacity, write, but do not solve, an equation using an integral expression to find how many hours it will take before the tank is completely full of sand.

$$\boxed{104.420 + \int_7^k S(t)\,dt = 200}$$

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Problem #2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$. $Y'(t)$

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 2+5\sin\left(\frac{4\pi t}{25}\right) = \boxed{31.816 \text{ cubic yards}}$

math 9

(b) $\boxed{Y(t) = 2500 + \int_0^t S(t)\,dt - \int_0^t R(t)\,dt}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = \frac{15(4)}{1+3(4)} - \left(2+5\sin\left(\frac{4\pi(4)}{25}\right)\right) = 4.615 - 6.524$

$= \boxed{-1.909 \text{ cubic yds/hr}}$

(d) Extreme Value Theorem

$Y'(t) = \underbrace{S(t) - R(t)}_{y_1} = \underbrace{0}_{y_2}$

$t = 5.118$ $\longrightarrow$

$Y(0) = 2500$

$Y(6) = 2500 + \int_0^6 S(t) - \int_0^6 R(t) = 2493.277$

$Y(5.118) = 2500 + \int_0^{5.118} S(t) - \int_0^{5.118} R(t) = 2492.369$

$\therefore$ The E.V.T guarantees the minimum amount of sand, 2492.369 pounds, will be when $t = 5.118$ hrs