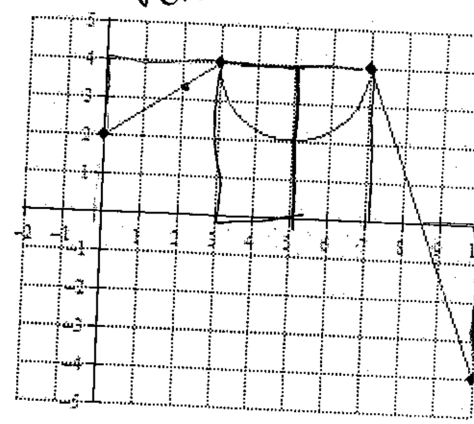
AP Calculus AB
Unit 6 – Day 5 – Assignment

Name: Answer Key *

The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the x – axis on the time interval $0 \le t \le 10$. The initial position of the particle at time $t = 0$ is 12.



1. On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

Right → $(0, 8.5)$ b/c $v(t) > 0$

left → $(8.5, 10)$ b/c $v(t) < 0$

2. What is the total distance that the particle has traveled on the time interval $0 \le t \le 7$. Leave your answer in terms of π. Indicate units of measure.

Total Distance $= \int_0^7 |v(t)|\,dt$

$= \frac{1}{2}(3)(2+4) + \left[4(4) - \frac{1}{2}\pi(2)^2\right]$

$= 9 + 16 - 2\pi = \boxed{25 - 2\pi \text{ meters}}$

3. What is the net distance that the particle travels on the interval $5 \le t \le 10$? Round your answer to the nearest thousandth. Indicate units of measure.

Net Distance $= \int_5^{10} v(t)\,dt = \left[(2)(4) - \frac{1}{4}\pi(2)^2\right] + \frac{1}{2}(2)(4) - \frac{1}{2}(2)(4)$

$= 8 - \pi + 4 - 4$

$= 8 - \pi = \boxed{4.858 \text{ meters}}$

4. What is the acceleration of the particle at time $t = 2$? Indicate units of measure.

$v'(2) = a(2) =$ slope of $v(t)$ at $t = 2$

$\boxed{\frac{2}{3} \text{ m/s}^2}$

5. What is the position of the particle at time $t = 5$? Indicate units of measure.

$\int_0^5 v(t)\,dt = p(5) - p(0)$

↓

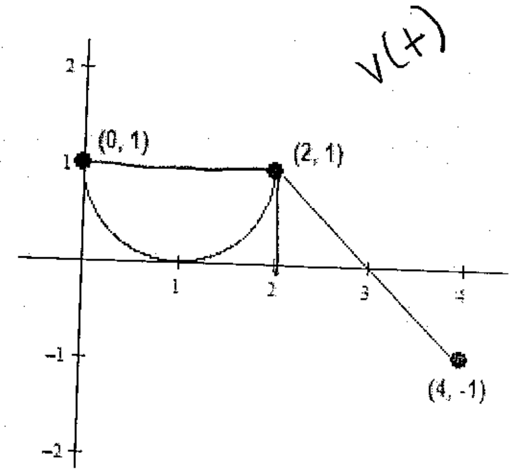
$\frac{1}{2}(3)(2+4) + \left[(2)(4) - \frac{1}{4}\pi(2)^2\right]$

$9 + 8 - \pi$

$17 - \pi = p(5) - 12$

$p(5) = \boxed{29 - \pi \text{ meters}}$

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the following questions.



6. For what values is the particle moving to the right? Justify your answer.

$(0,1) \cup (1,3)$ b/c $v(t) > 0$

7. For what values is the particle moving to the left? Justify your answer.

$(3,4)$ b/c $v(t) < 0$

8. For what values is the speed of the particle increasing? Justify your answer.

$v(t)$: + + + (0 to 1), + + + (1 to 2), + + + (2 to 3), − − − (3 to 4)

$a(t)$: − − − (0 to 1), + + + + (1 to 2), − − − (2 to 3), − − − (3 to 4)

$(1,2) \cup (3,4)$ b/c $v(t)$ and $a(t)$ have same sign.

9. For what values is the speed of the particle decreasing? Justify your answer.

$(0,1) \cup (2,3)$ b/c $v(t)$ and $a(t)$ have different signs.

10. What is the net distance that the particle travels on the interval [0, 4]?

$$\int_0^4 v(t)\,dt = \left[(2)(1) - \tfrac{1}{2}\pi(1)^2\right] + \tfrac{1}{2}\cancel{(1)}(1) - \tfrac{1}{2}\cancel{(1)(1)}$$

$$\boxed{2 - \tfrac{1}{2}\pi}$$

11. What is the total distance that the particle travels on the interval [0, 4]?

$$\int_0^4 |v(t)|\,dt = \left[2(1) - \tfrac{1}{2}\pi(1)^2\right] + \tfrac{1}{2}(1)(1) + \tfrac{1}{2}(1)(1)$$

$$= 2 - \tfrac{1}{2}\pi + \tfrac{1}{2} + \tfrac{1}{2}$$

$$= \boxed{3 - \tfrac{1}{2}\pi}$$

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous and differentiable functions on $0 \le t \le 60$. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec²)	1	5	2	1	2	4	2

12. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)|dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

$\int_0^{60} |v(t)|dt \approx 25(30) + 10(14) + 25(0) \approx \boxed{890\text{ ft}}$

Total distance the car traveled from 0 to 60 seconds.

13. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t)dt$ in terms of the car's motion. Find the exact value of the integral.

$\int_{15}^{50} a(t)\,dt = v(50) - v(15) = 0 - (-30) = \boxed{30\text{ ft/sec}}$

Represents the change in velocity b/t 15 and 50 seconds.

14. Is there a value of t such that $a'(t) = 0$? If so, identify an interval on which such a value of t exists? Justify your reasoning.

Since $a(0) = 1$ & $a(30) = 1$, Rolle's Thm guarantees a value of t on $\boxed{(0,30)}$ such that $a'(t) = 0$

$\boxed{\text{OR } (25,35) \text{ or } (35,60)}$

15. Using appropriate units, approximate the value of $v'(31)$. What does this value say about the motion of the car at $t = 31$.

$v'(31) \approx \dfrac{v(30) - v(35)}{30 - 35} = \dfrac{-14 - (-10)}{-5} \approx \boxed{4/5\text{ ft/sec}^2}$

Since $v'(31) > 0$, then the velocity is increasing at $t = 31$ sec.

60-25

16. Using appropriate units, find the value and explain the meaning of $\frac{1}{35}\int_{25}^{60} a(t)dt$.

$\frac{1}{35}\int_{25}^{60} a(t)dt = \frac{1}{35}[v(60) - v(25)] = \frac{1}{35}[10 - (-20)] = \boxed{\frac{6}{7}\text{ ft/sec}}$

Represents the average acceleration of the car from $t = 25$ to $t = 60$ seconds.

(calc Active)

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Problem #3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

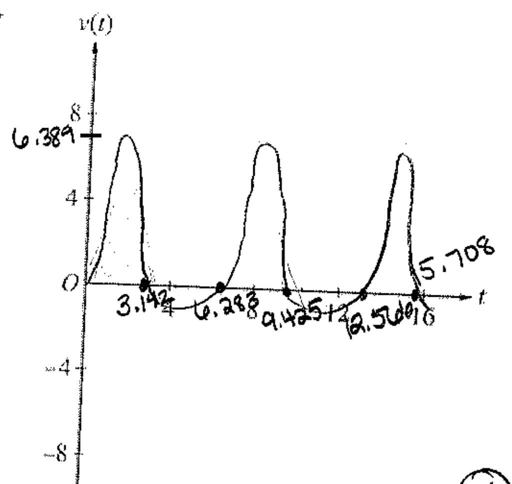
(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

$v(t)$



(b) left when $v(t) < 0$

$(3.142, 6.283)$ & $(9.425, 12.566)$ & $(15.708, 16)$

(c) $\int_0^4 |v(t)|\,dt = \boxed{10.542}$

math 9

(d) put $\int_0^{16} v(t)\,dt$ in your $y=$ & graph.

$\int_0^{16} v(t)\,dt > 0$, so there is no place where the particle returns to the origin.