5.3 Sum & Difference Identities

ESSENTIAL QUESTION:
How are sum and difference identities used to find trig functions of odd-ball angles (like sin 75°)?

SUM AND DIFFERENCE IDENTITIES

$\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$ "opposite signs"

$\underset{-}{+}$ $\underset{+}{-}$

$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$

"Same signs"

$\tan(u \pm v) = \dfrac{\tan(u) \pm \tan(v)}{1 \mp \tan(u)\tan(v)}$ → "same sign" ↘ "opposite sign"

$\frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

*Put on different slides!

examples...

Use a sum or difference identity to find an exact value:

1. cos 75°

$\cos(45° + 30°)$

$\cos 45° \cos 30° - \sin 45° \sin 30°$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

2. $\sin\frac{\pi}{12}$

3. $\tan\frac{11\pi}{12}$ $\tan(165°)$

$\tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \dfrac{\tan\left(\frac{2\pi}{3}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}$

$(120° + 45°)$

$\frac{8\pi}{12} + \frac{3\pi}{12}$

$\frac{2\pi}{3} + \frac{\pi}{4}$

$= \dfrac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{1 - 2\sqrt{3} + 3}{1 - 3} = \dfrac{4 - 2\sqrt{3}}{-2}$

$\cancel{\frac{-2(-2+\sqrt{3})}{}} = \boxed{\sqrt{3} - 2}$

Write each expression in terms of a single angle.

4. cos 94° cos 18° + sin 94° sin18°

$\cos(94° - 18°)$

$\boxed{\cos(76°)}$

5. $\sin\frac{\pi}{7}\cos\frac{\pi}{3} - \cos\frac{\pi}{7}\sin\frac{\pi}{3}$

$\sin\left(\frac{\pi}{7} - \frac{\pi}{3}\right)$

$\sin\left(\frac{3\pi}{21} - \frac{7\pi}{21}\right) = \boxed{\sin\left(-\frac{4\pi}{21}\right)}$

Use sum or difference identities to verify the identity.

6. $\boxed{\sin\left(\frac{\pi}{2} - \theta\right)} = \cos\theta$

$$= \sin\left(\frac{\pi}{2}\right)\cos(\theta) - \cos\left(\frac{\pi}{2}\right)\sin(\theta)$$

$$\downarrow \qquad \downarrow$$

$$= (1)\cos(\theta) - (0)\sin(\theta)$$

$$= \boxed{\cos\theta} \quad \checkmark$$

7. $\boxed{\cos\left[\left(\frac{\pi}{2} - x\right) - y\right]} = \sin(x + y)$

$$= \underbrace{\cos\left(\frac{\pi}{2} - x\right)}\cos(y) + \underbrace{\sin\left(\frac{\pi}{2} - x\right)}\sin(y)$$

$$= \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$= \boxed{\sin(x+y)} \quad \checkmark$$