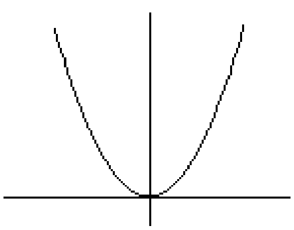
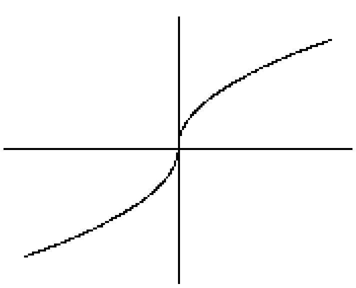
AP Calculus
Unit 6 – Basic Integration & Applications

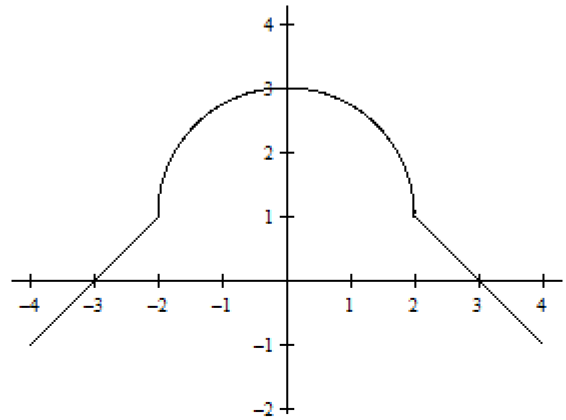
Day 4 Notes: Properties of Definite Integrals

1. $\int_a^a f(x)dx =$
2. Given that $a < c < b$, $\int_a^b f(x)dx =$
3. If $\int_a^b f(x)dx = K$, then $\int_b^a f(x)dx =$
4. Given that $b < a$, then $\int_a^b f(x)dx =$
5. If k is a constant, then $\int_a^b k \cdot f(x)dx =$
6. $\int_a^b [f(x) \pm g(x)]dx =$
7. Given that $f(x)$ is an even function, $\int_{-a}^a f(x)dx =$ 
8. Given that $f(x)$ is an odd function, $\int_{-a}^a f(x)dx =$ 

If $\int_0^3 f(x)dx = 6$ and $\int_3^7 f(x)dx = -8$, determine the value of each of the following integrals using the properties of definite integrals. Explain how you arrived at your answer for each.

$\int_3^0 f(x)dx$	$\int_0^7 f(x)dx$
$\int_3^3 f(x)dx$	$\int_7^3 3f(x)dx$
$\int_3^7 (2+3f(x))dx$	$\int_{-3}^3 f(x)dx$, if *f*(*x*) is an even function
$\int_{-3}^3 f(x)dx$, if *f*(*x*) is an odd function	

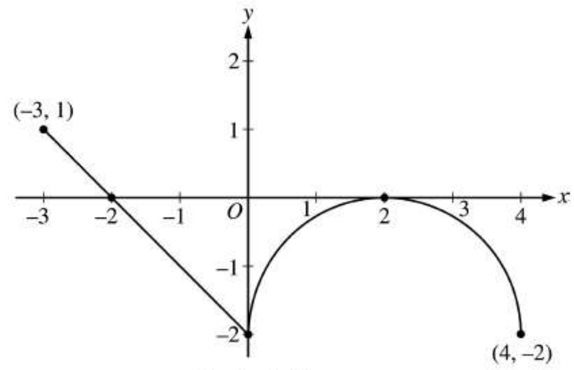
Pictured to the right is the graph of a function $f(x)$.



What is the value of $\int_0^3 f(x)dx$?	
What is the value of $\int_0^4 f(x)dx$?	What is the value of $\int_{-3}^3 f(x)dx$?
If $F(0) = 5$, what is the value of $F(3)$, where F is the anti-derivative of $f(x)$?	If $F(-2) = -2$, what is the value of $F(2)$, where F is the anti-derivative of $f(x)$?

2003 AP® CALCULUS AB

Problem #4



Graph of f'

Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

a. On what intervals, if any, is f increasing. Justify your reasoning.

b. Find the x – coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$.
Justify your answer.

c. Find an equation for the line tangent to the graph of f at the point (0, 3).

d. Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

AP Calculus AB **Name: ______________________________**
Unit 6 – Day 4 – Assignment

Given $\int_2^6 f(x)dx = 10$ and $\int_2^6 g(x)dx = -2$, find the values of each of the following definite integrals, if possible, by rewriting the given integral using the properties of integrals.

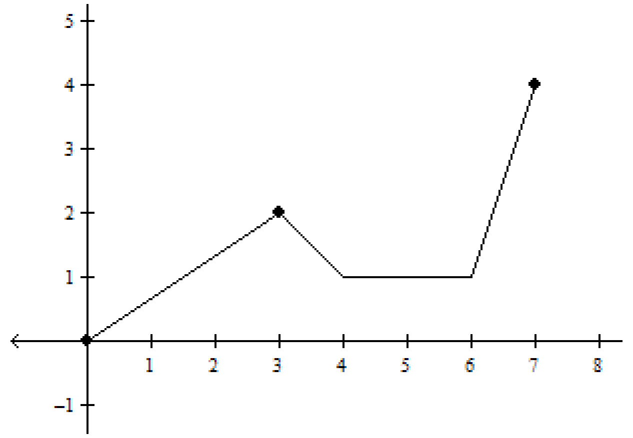
1. $\int_2^6 [f(x) + g(x)]dx$	2. $\int_2^6 [2f(x) - 3g(x)]dx$	3. $\int_2^6 2x + 2g(x)dx$

Given $\int_{-2}^4 f(x)dx = -6$ and $\int_{-2}^4 g(x)dx = 4$, find the values of each of the following definite integrals. Rewrite the given integral using the properties of integrals. Then, find the value.

4. $\int_{-2}^4 [f(x) + 4]dx$	5. $\int_{-2}^4 [3g(x) + x]dx$
6. $\int_{-2}^4 \left[\frac{1}{2} f(x) + 3x^2\right]dx$	

Pictured below is the graph of $f'(x)$, the first derivative of a function *f*(*x*). Use the graph to answer the following questions 8 –10.

Graph of $f'(x)$



7. What is the value of $\int_0^7 f'(x)dx$

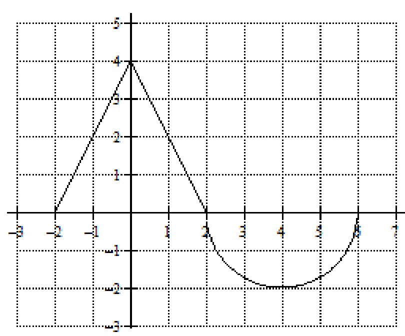
8. If *f*(0) = –3, what is the value of *f*(3)?

9. If *f*(3) = –1, what is the value of *f*(7)?

The graph of $f'(x)$, the derivative of a function, *f*(*x*), is pictured below on the interval [–2, 6]. Write and find the value of a definite integral to find each of the indicated values of *f*(*x*) below. Also, *f*(–2) = 5.

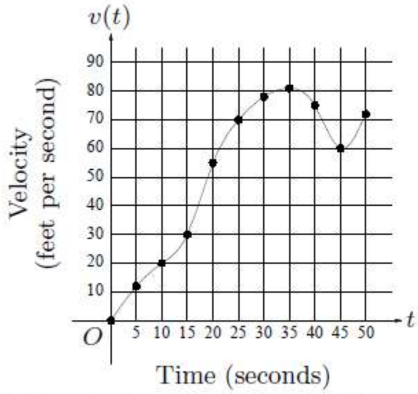
10. Find the value of *f*(0).

11. Find the value of *f*(6).



Graph of f '

1998 Calculus AB



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

1999 AP Calculus AB

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24–hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.