AP Calculus AB
Unit 6 – Day 4 – Assignment

Name: Answer Key *

Given $\int_2^6 f(x)dx = 10$ and $\int_2^6 g(x)dx = -2$, find the values of each of the following definite integrals, if possible, by rewriting the given integral using the properties of integrals.

1. $\int_2^6 [f(x) + g(x)]dx$

$= \int_2^6 f(x)dx + \int_2^6 g(x)dx$

$= 10 + -2$

$= \boxed{8}$

2. $\int_2^6 [2f(x) - 3g(x)]dx$

$= 2\int_2^6 f(x)dx - 3\int_2^6 g(x)dx$

$= 2(10) - 3(-2)$

$= 20 + 6$

$= \boxed{26}$

3. $\int_2^6 2x + 2g(x)dx$

$= \int_2^6 2x\,dx + 2\int_2^6 g(x)dx$

$= \frac{\cancel{2}x^2}{\cancel{2}}\Big]_2^6 + 2(-2)$

$= [(6)^2] - [(2)^2] + -4$

$= 36 - 4 - 4 = \boxed{28}$

Given $\int_{-2}^4 f(x)dx = -6$ and $\int_{-2}^4 g(x)dx = 4$, find the values of each of the following definite integrals. Rewrite the given integral using the properties of integrals. Then, find the value.

4. $\int_{-2}^4 [f(x) + 4]dx$

$\int_{-2}^4 f(x)dx + \int_{-2}^4 4\,dx$

$-6 + 4x\Big]_{-2}^4$

$-6 + [4(4)] - [4(-2)]$

$-6 + 16 + 8 = \boxed{18}$

5. $\int_{-2}^4 [3g(x) + x]dx$

$3\int_{-2}^4 g(x)dx + \int_{-2}^4 x\,dx$

$3(4) + \frac{1}{2}x^2\Big]_{-2}^4$

$12 + [\frac{1}{2}(4)^2] - [\frac{1}{2}(-2)^2]$

$12 + 8 + -2 = \boxed{18}$

6. $\int_{-2}^4 \left[\frac{1}{2}f(x) + 3x^2\right]dx$

$\frac{1}{2}\int_{-2}^4 f(x)dx + \int_{-2}^4 3x^2dx$

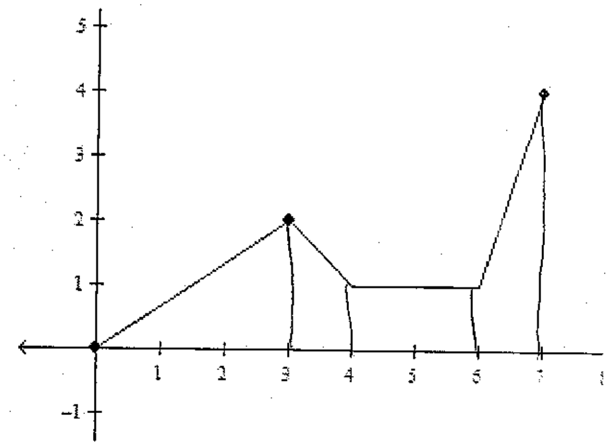
$\frac{1}{2}(-6) + \frac{\cancel{3}x^3}{\cancel{3}}\Big]_{-2}^4$

$-3 + [(4)^3] - [(-2)^3]$

$-3 + 64 + 8 = \boxed{69}$

Pictured below is the graph of $f'(x)$, the first derivative of a function $f(x)$. Use the graph to answer the following questions 8 – 10.

Graph of $f'(x)$



7. What is the value of $\int_0^7 f'(x)dx$

$= \frac{1}{2}(3)(2) + \frac{1}{2}(1)(2+1) + (2)(1) + \frac{1}{2}(1)(1+4)$

$= 3 + 1.5 + 2 + 2.5$

$= \boxed{9}$

8. If $f(0) = -3$, what is the value of $f(3)$?

$\int_0^3 f'(x)dx = f(3) - f(0)$

$\downarrow$

$\frac{1}{2}(3)(2) = f(3) - (-3)$

$3 = f(3) + 3$

$\boxed{f(3) = 0}$

9. If $f(3) = -1$, what is the value of $f(7)$?

$\int_3^7 f'(x)dx = f(7) - f(3)$

$\downarrow$

$\underbrace{\frac{1}{2}(1)(2+1) + (2)(1) + \frac{1}{2}(1)(1+4)} = f(7) - (-1)$

$1.5 + 2 + 2.5$

$6 = f(7) + 1$

$\boxed{5 = f(7)}$

The graph of $f'(x)$, the derivative of a function, $f(x)$, is pictured below on the interval $[-2, 6]$. Write and find the value of a definite integral to find each of the indicated values of $f(x)$ below. Also, $f(-2) = 5$.

10. Find the value of $f(0)$.

$\int_{-2}^0 f'(x)dx = f(0) - f(-2)$

$\downarrow$

$\frac{1}{2}(2)(4) = f(0) - 5$

$4 = f(0) - 5$

$\boxed{f(0) = 9}$

11. Find the value of $f(6)$.

$\int_{-2}^6 f'(x)dx = f(6) - f(-2)$

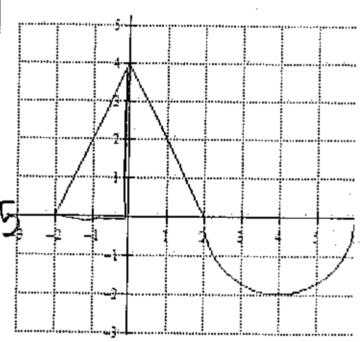
$\downarrow$

$\underbrace{\frac{1}{2}(2)(4) + \frac{1}{2}(2)(4) - \frac{1}{2}\pi(2)^2} = f(6) - 5$

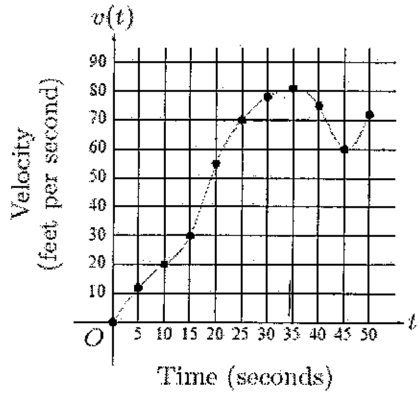
$4 + 4 - 2\pi$

$8 - 2\pi = f(6) - 5$

$\boxed{13 - 2\pi = f(6)}$



Graph of f'



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a) $a(t) > 0$ when $v(t)$ is increasing.

$\boxed{(0,35) \text{ \& } (45,50)}$

(b) avg accleration $= \dfrac{v(0)-v(50)}{0-50} = \dfrac{0-72}{-50} = \boxed{1.44 \text{ ft/sec}^2}$

(c) accleration at $t=40 \to \dfrac{v(35)-v(45)}{35-45} = \dfrac{81-60}{-10} \approx \boxed{-2.1 \text{ ft/sec}^2}$

(d) $\int_0^{50} v(t)\,dt = 10(12) + 10(30) + 10(70) + 10(81) + 10(60)$

area of Rectangles

$= 120 + 300 + 700 + 810 + 600$

$= \boxed{2530 \text{ ft}}$

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3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.
   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

ⓐ $\int_0^{24} R(t)\,dt = 6(10.4) + 6(11.2) + 6(11.3) + 6(10.2)$

Rectangle area

$= 62.4 + 67.2 + 67.8 + 61.2$

$= \boxed{258.6 \text{ gallons}}$

This is an approximation to the total flow in gallons of water from the pipe in the 24-hour period.

ⓑ $R'(t) = 0$

Rolle's Theorem

① $R(t)$ is differentiable & continuous ✓

② $R(0) = 9.6$, $R(24) = 9.6$ } $R(0) = R(24)$ ✓

∴ According to Rolle's Theorem, there is such a t on $0 < t < 24$ such that $R'(t) = 0$