AP Calculus
Unit 6 – Basic Integration & Applications

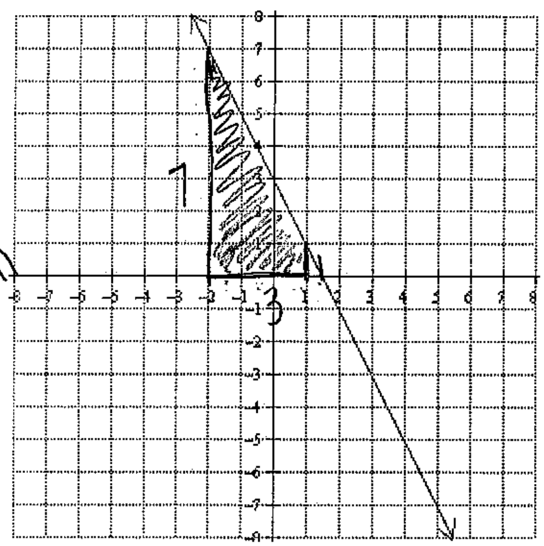
Day 3 Notes: Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus, Part I

If $f(x)$ is continuous on $[a,b]$ and $F(x)$ is the antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Consider the function $f(x) = -2x + 3$ whose graph is pictured below. Calculate each of the following definite integrals according to the Fundamental Theorem of Calculus. Then, shade the area of the region that the integral represents.



Trapezoid
$A = \frac{1}{2}(3)(7+1)$
$A = 12$ ✓

Find $\int_{-2}^{1}(-2x+3)dx$.

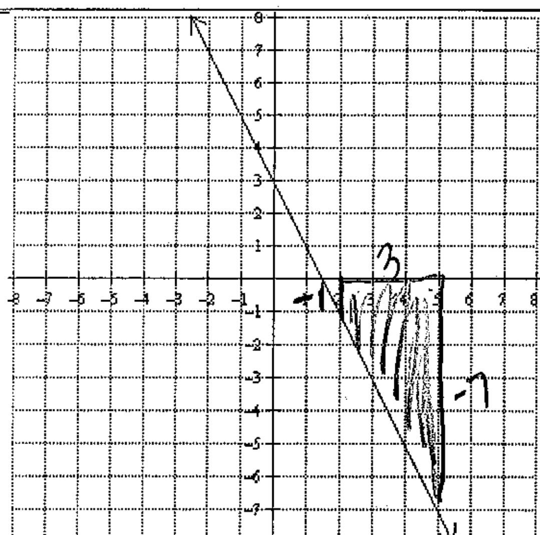
$$= \frac{-2x^2}{2} + \frac{3x}{1} + C \Big]_{-2}^{1}$$

$$= -x^2 + 3x + C \Big]_{-2}^{1}$$

$$= [-(1)^2 + 3(1) + C] - [-(-2)^2 + 3(-2) + C]$$

$$= [-1 + 3 + C] - [-4 - 6 + C]$$

$$= -1 + 3 + \cancel{C} + 4 + 6 - \cancel{C} = \boxed{12}$$



$A = \frac{1}{2}(3)(-1 + -7)$

$A = -12$ ✓

Find $\int_{2}^{5}(-2x+3)dx$.

$$= \frac{-2x^2}{2} + \frac{3x}{1} + C \Big]_{2}^{5}$$

$$= -x^2 + 3x + C \Big]_{2}^{5}$$

$$= [-(5)^2 + 3(5) + C] - [-(2)^2 + 3(2) + C]$$

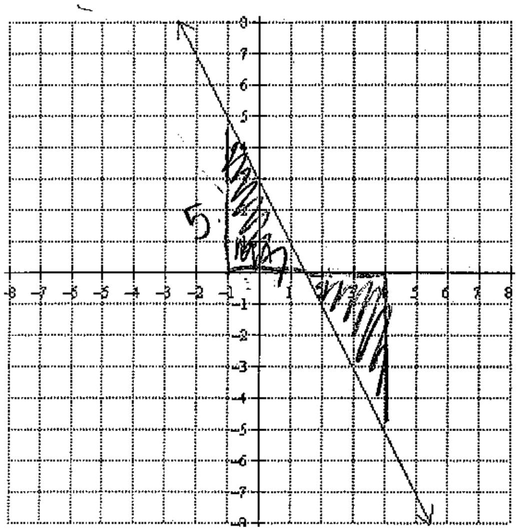
$$= -25 + 15 + \cancel{C} + 4 - 6 - \cancel{C}$$

$$= \boxed{-12}$$

$A = \frac{1}{2}(5)(2.5) = 6.25$

$A = \frac{1}{2}(-5)(2.5) = -6.25$

$6.25 + -6.25 = 0$ ✓



Find $\int_{-1}^{4}(-2x+3)dx$. $= -x^2+3x+C\Big]_{-1}^{4}$

$= \left[-(4)^2+3(4)+C\right] - \left[-(-1)^2+3(-1)+C\right]$

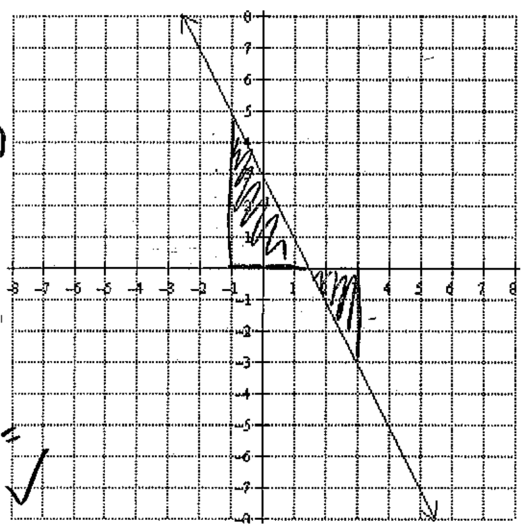
$= -16+12+\cancel{C}+1+3-\cancel{C}$

$= \boxed{0}$

$A = \frac{1}{2}(5)(2.5)$
$= 6.25$

$A = \frac{1}{2}(-3)(1.5)$
$= -2.25$

$6.25 - 2.25 = 4$ ✓



Find $\int_{-1}^{3}(-2x+3)dx$. $= -x^2+3x+C\Big]_{-1}^{3}$

$= \left[-(3)^2+3(3)+C\right] - \left[-(-1)^2+3(-1)+C\right]$

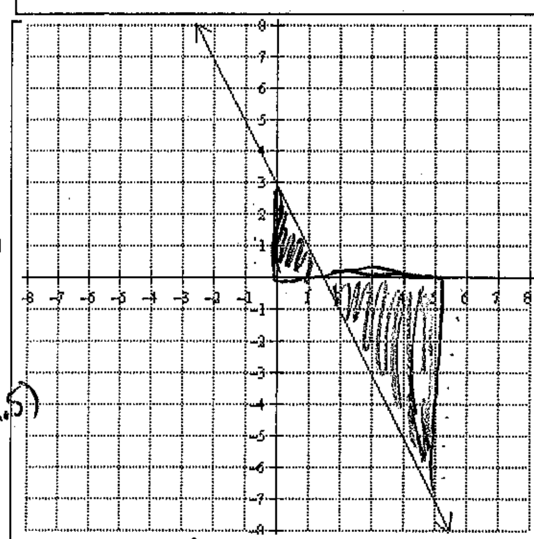
$= -9+9+\cancel{C}+1+3-\cancel{C}$

$= \boxed{4}$

$A = \frac{1}{2}(3)(1.5)$
$= 2.25$

$A = \frac{1}{2}(-7)(3.5)$
$= -12.25$

$-12.25 + 2.25 = -10$ ✓



Find $\int_{0}^{5}(-2x+3)dx$. $= -x^2+3x+C\Big]_{0}^{5}$

$= \left[-(5)^2+3(5)+C\right] - \left[-(0)^2+3(0)+C\right]$

$= -25+15+\cancel{C}-\cancel{0}-\cancel{0}-C$

$= \boxed{-10}$

Based on the results of the five previous examples, what inferences can you make about the value of the definite integral and the amount of area bounded by the graph of the integrand and the x – axis?

$\int_a^b f(x)\,dx$ = the area of the region bounded by f(x), the x-axis, x=a and x=b.

Find each of the following definite integrals applying the fundamental theorem of calculus. Show your work. Then, use your graphing calculator to verify your results.

Cont ✓

$$\int_{-1}^{2}(2x-x^2)\,dx = \frac{2x^2}{2} - \frac{x^3}{3} + C\Big]_{-1}^{2}$$

$$= x^2 - \tfrac{1}{3}x^3 + C\Big]_{-1}^{2}$$

$$= \left[(2)^2 - \tfrac{1}{3}(2)^3 + C\right] - \left[(-1)^2 - \tfrac{1}{3}(-1)^3 + C\right]$$

$$= 4 - \frac{8}{3} + \cancel{C} - 1 - \frac{1}{3} - \cancel{C}$$

$$= \boxed{0}$$

$$\int_{-2}^{3}\left(\frac{1}{x^2}+x\right)dx$$

D.N.E b/c FTC is not applicable since the integrand is not continuous on [-2,3] as x=0 is a discontinuity.

only discont at x=0

$$\int_1^4 \left(\frac{2x+3}{x^2}\right)dx = \int_1^4 2x^{-1} + 3x^{-2}\,dx$$

$$= \frac{2x^0}{0} + \frac{3x^{-1}}{-1} + C\Big]_1^4$$

$$= 2\ln x - \frac{3}{x} + C\Big]_1^4$$

$$= \left[2\ln 4 - \tfrac{3}{4} + C\right] - \left[2\ln 1 - \tfrac{3}{1} + C\right]$$

$$= 2\ln 4 - \frac{3}{4} + \cancel{C} - \cancel{0} + 3 - \cancel{C}$$

$$= \boxed{2\ln 4 + \frac{9}{4}}$$

$$\int_0^{\pi}(2x+\cos x)\,dx = \frac{2x^2}{2} + \sin x + C\Big]_0^{\pi}$$

$$= x^2 + \sin x + C\Big]_0^{\pi}$$

$$= \left[(\pi)^2 + \sin\pi + C\right] - \left[(0)^2 + \sin 0 + C\right]$$

$$= \pi^2 + \cancel{0} + \cancel{C} - \cancel{0} + 0 - \cancel{C}$$

$$= \boxed{\pi^2}$$

Pictured below is a table of values that shows the values of a function, $f(x)$, and its first and second derivative for selected values of x. Use the information in the table to answer the questions that follow.

x	-3	-1	1	3	5
$f(x)$	4	0	-2	1	3
$f'(x)$	-2	1	0	3	2
$f''(x)$	-1	0	2	-3	-1

1. What is the value of $\int_{-3}^{1} f'(x)dx$.

$\int_{-3}^{1} f'(x)\,dx = f(1) - f(-3)$

$= -2 - 4 = \boxed{-6}$

2. What is the value of $\int_{-1}^{3} f'(x) + f''(x)dx$?

$= f(x) + f'(x)\Big]_{-1}^{3}$

$= [f(3) + f'(3)] - [f(-1) + f'(-1)]$

$= 1 + 3 + 0 - 1 = \boxed{3}$

3. What is the value of $\int_{1}^{5} 3f''(x)dx$?

$= 3f'(x)\Big]_{1}^{5}$

$= [3f'(5)] - [3f'(1)]$

$= 3(2) - 3(0) = \boxed{6}$

4. What is the value of $\int_{-3}^{3} \frac{1}{2}f'(x) + 2f''(x)dx$?

$\frac{1}{2}f(x) + 2f'(x)\Big]_{-3}^{3}$

$= [\frac{1}{2}f(3) + 2f'(3)] - [\frac{1}{2}f(-3) + 2f'(-3)]$

$= \frac{1}{2}(1) + 2(3) - \frac{1}{2}(4) - 2(-2) = \frac{1}{2} + 6 - 2 + 4 = \boxed{8.5}$

5. What is the equation of the tangent line to the graph of $f(x)$ at $x = 3$?

$f(3) = 1$

$f'(3) = 3$

$\boxed{y - 1 = 3(x - 3)}$

6. Use the equation of the tangent line in #5 to approximate the value of $f(3.1)$. Is this an over or under approximation of $f(3.1)$? Give a reason for your answer.

$y - 1 = 3(3.1 - 3)$

$y - 1 = 3(.1)$

$y - 1 = .3$

$y = \boxed{1.3}$

Since $f''(3) = -3 < 0$, then $f(x)$ is concave down at $x = 3$ which means this is an over approximation.