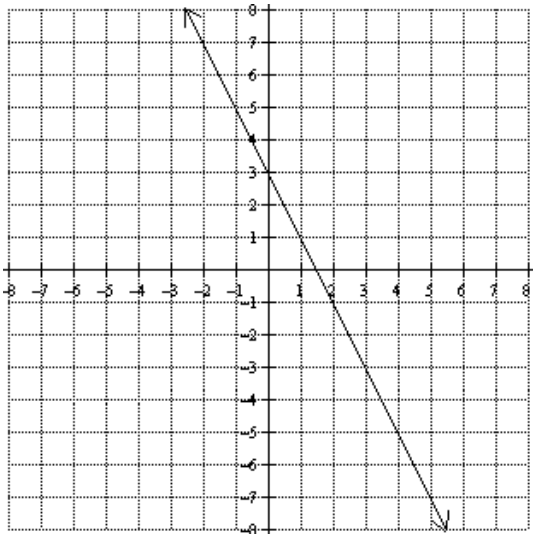
AP Calculus
Unit 6 – Basic Integration & Applications

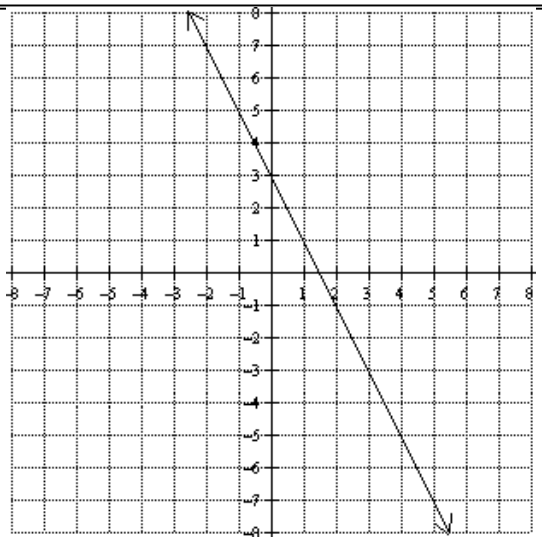
Day 3 Notes: Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus, Part I

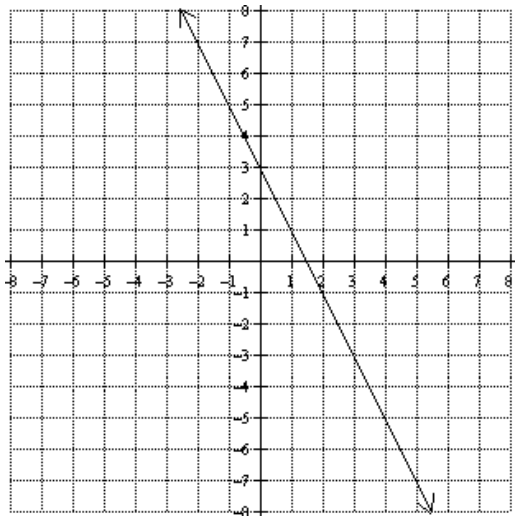
Consider the function $f(x) = -2x + 3$ whose graph is pictured below. Calculate each of the following definite integrals according to the Fundamental Theorem of Calculus. Then, shade the area of the region that the integral represents.



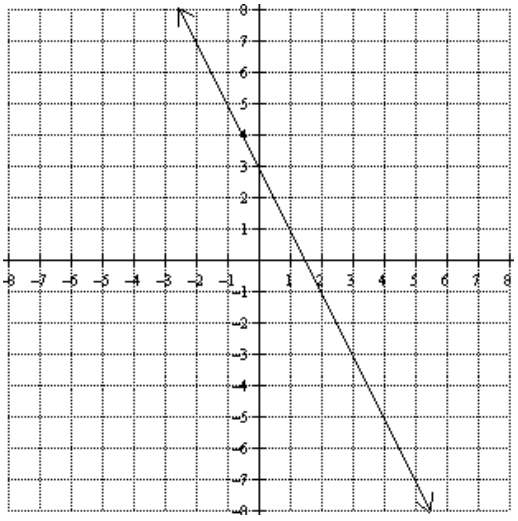
Find $\int_{-2}^{1}(-2x+3)dx$.



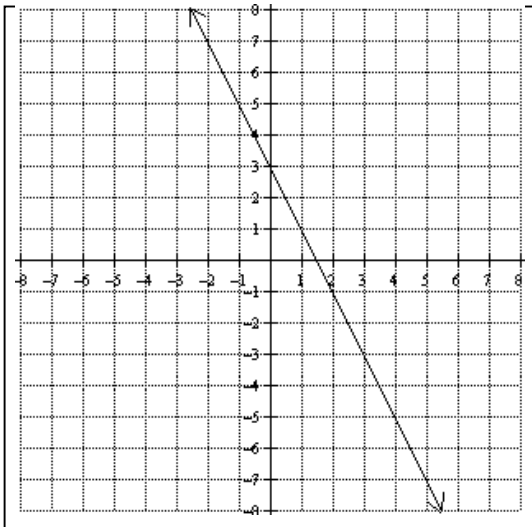
Find $\int_{2}^{5}(-2x+3)dx$.



Find $\int_{-1}^{4}(-2x+3)dx$.



Find $\int_{-1}^{3}(-2x+3)dx$.



Find $\int_{0}^{5}(-2x+3)dx$.

Based on the results of the five previous examples, what inferences can you make about the value of the definite integral and the amount of area bounded by the graph of the integrand and the x – axis?

Find each of the following definite integrals applying the fundamental theorem of calculus. Show your work. Then, use your graphing calculator to verify your results.

$\int_{-1}^{2}\left(2x-x^2\right)dx$	$\int_{-2}^{3}\left(\frac{1}{x^2}+x\right)dx$
$\int_{1}^{4}\left(\frac{2x+3}{x^2}\right)dx$	$\int_{0}^{\pi}\left(2x+\cos x\right)dx$

Pictured below is a table of values that shows the values of a function, $f(x)$, and its first and second derivative for selected values of x. Use the information in the table to answer the questions that follow.

x	−3	−1	1	3	5
$f(x)$	4	0	−2	1	3
$f'(x)$	−2	1	0	3	2
$f''(x)$	−1	0	2	−3	−1

1. What is the value of $\int_{-3}^{1} f'(x)dx$.

2. What is the value of $\int_{-1}^{3} f'(x) + f''(x)dx$?

3. What is the value of $\int_{1}^{5} 3f''(x)dx$?

4. What is the value of $\int_{-3}^{3} \frac{1}{2} f'(x) + 2f''(x)dx$?

5. What is the equation of the tangent line to the graph of $f(x)$ at $x = 3$?

6. Use the equation of the tangent line in #5 to approximate the value of $f(3.1)$. Is this an over or under approximation of $f(3.1)$? Give a reason for your answer.

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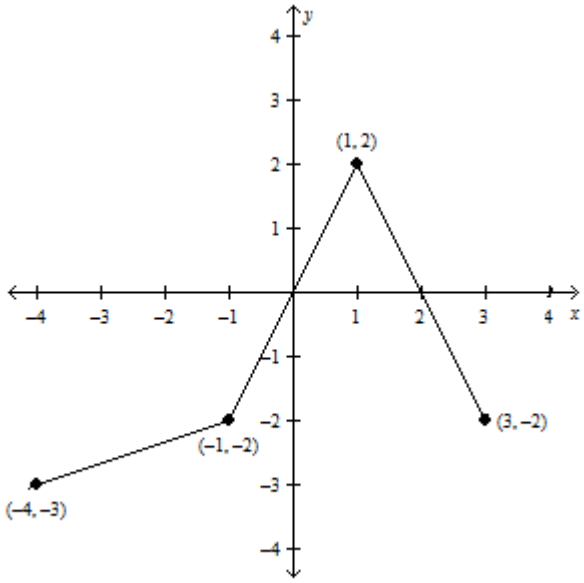
Unit 6 – Day 3 – Assignment

For exercises 1 – 6, find the value of the definite integral. Show your algebraic work.

1. $\int_{-1}^{1}\left(t^2 - t\right)dt$	2. $\int_{1}^{2}\left(\frac{3}{x^2} - 1\right)dx$
3. $\int_{1}^{4}\frac{u-2}{\sqrt{u}}\,du$	4. $\int_{-2}^{-1}\left(x - \frac{1}{x^2}\right)dx$
5. $\int_{0}^{\pi}(1+\sin x)dx$	6. $\int_{1}^{3}\left(3x^2 + 5x - 4\right)dx$

Pictured to the right is the graph of a function *f*. In exercises 7 – 12, find the values of each of the following definite integrals. If a value does not exist, explain why.



7. $\int_{1}^{2} f(x)dx$	8. $\int_{0}^{3} f(x)dx$	9. $\int_{-1}^{1} f(x)dx$
10. $\int_{-4}^{0} f'(x)dx$	11. $\int_{-1}^{1} f'(x)dx$	12. $\int_{1}^{3} f'(x)dx$

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Question 5

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

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Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.