AP Calculus AB
Unit 6 – Day 3 – Assignment

Name: Answer Key*

For exercises 1 – 6, find the value of the definite integral. Show your algebraic work.

cont.

1. $\int_{-1}^{1}(t^2 - t)\,dt$

$$= \frac{t^3}{3} - \frac{t^2}{2} + C\Big]_{-1}^{1}$$
$$= \frac{1}{3}t^3 - \frac{1}{2}t^2 + C\Big]_{-1}^{1}$$
$$= \left[\tfrac{1}{3}(1)^3 - \tfrac{1}{2}(1)^2 + C\right] - \left[\tfrac{1}{3}(-1)^3 - \tfrac{1}{2}(-1)^2 + C\right]$$
$$= \frac{1}{3} - \cancel{\frac{1}{2}} + \cancel{C} + \frac{1}{3} + \cancel{\frac{1}{2}} - \cancel{C}$$
$$= \boxed{\frac{2}{3}}$$

2. $\int_{1}^{2}\left(\frac{3}{x^2} - 1\right)dx$

discont. at $x=0$

$$= \int_1^2 3x^{-2} - 1\,dx$$
$$= \frac{3x^{-1}}{-1} - \frac{1x}{1} + C\Big]_1^2$$
$$= -\frac{3}{x} - x + C\Big]_1^2$$
$$\left[-\frac{3}{2} - 2 + C\right] - \left[-\frac{3}{1} - 1 + C\right]$$
$$= -\frac{3}{2} - 2 + \cancel{C} + 3 + 1 - \cancel{C} = \boxed{\frac{1}{2}}$$

discont. at $x=0$

3. $\int_{1}^{4}\frac{u-2}{\sqrt{u}}\,du$ ($u^{1/2}$)

$$= \int_1^4 u^{1/2} - 2u^{-1/2}\,du$$
$$= \frac{u^{3/2}}{\frac{3}{2}} - \frac{2u^{1/2}}{\frac{1}{2}} + C\Big]_1^4$$
$$= \frac{2}{3}\sqrt{u^3} - 4\sqrt{u} + C\Big]_1^4$$
$$= \left[\tfrac{2}{3}\sqrt{4^3} - 4\sqrt{4} + C\right] - \left[\tfrac{2}{3}\sqrt{1^3} - 4\sqrt{1} + C\right]$$
$$= \frac{16}{3} - 8 + \cancel{C} - \frac{2}{3} + 4 - \cancel{C} = \boxed{\frac{2}{3}}$$

4. $\int_{-2}^{-1}\left(x - \frac{1}{x^2}\right)dx$

discont. at $x=0$

$$= \int_{-2}^{-1} x - x^{-2}\,dx$$
$$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + C\Big]_{-2}^{-1}$$
$$= \frac{1}{2}x^2 + \frac{1}{x} + C\Big]_{-2}^{-1}$$
$$= \left[\tfrac{1}{2}(-1)^2 + \tfrac{1}{-1} + C\right] - \left[\tfrac{1}{2}(-2)^2 + \tfrac{1}{-2} + C\right]$$
$$= \frac{1}{2} - 1 + \cancel{C} - 2 + \frac{1}{2} - \cancel{C} = \boxed{-2}$$

cont.

5. $\int_{0}^{\pi}(1 + \sin x)\,dx$

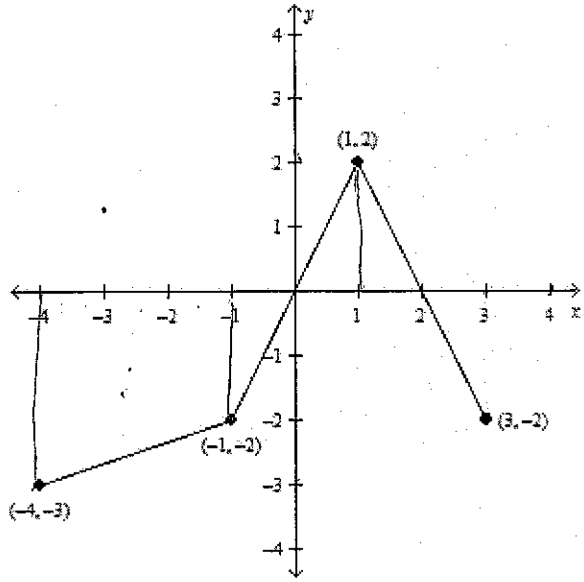
$$= \frac{1x}{1} - \cos x + C\Big]_0^{\pi}$$
$$= \left[\pi - \cos\pi + C\right] - \left[0 - \cos 0 + C\right]$$
$$= \pi - \underline{(-1)} + \cancel{C} - \cancel{0} + 1 - \cancel{C}$$
$$\boxed{\pi + 2}$$

6. $\int_{1}^{3}(3x^2 + 5x - 4)\,dx$

cont.

$$= \frac{3x^3}{3} + \frac{5x^2}{2} - \frac{4x}{1} + C\Big]_1^3$$
$$= x^3 + \frac{5}{2}x^2 - 4x + C\Big]_1^3$$
$$= \left[(3)^3 + \tfrac{5}{2}(3)^2 - 4(3) + C\right] - \left[(1)^3 + \tfrac{5}{2}(1)^2 - 4(1) + C\right]$$
$$= 27 + \frac{45}{2} - 12 + \cancel{C} - 1 - \frac{5}{2} + 4 - \cancel{C}$$
$$\boxed{38}$$

Pictured to the right is the graph of a function f. In exercises 7 – 12, find the values of each of the following definite integrals. If a value does not exist, explain why.



7. $\int_{1}^{2} f(x)dx$ $\frac{1}{2}(1)(2) = \boxed{1}$	8. $\int_{0}^{3} f(x)dx$ $\frac{1}{2}(2)(2) + \frac{1}{2}(1)(-2)$ $2 + -1$ $\boxed{1}$	9. $\int_{-1}^{1} f(x)dx$ $\frac{1}{2}(1)(-2) + \frac{1}{2}(1)(2)$ $-1 + 1 = \boxed{0}$
10. $\int_{-4}^{0} f'(x)dx$ D.N.E. b/c $f'(x)$ is not continuous at $x=-1$ as $f(x)$ is not differentiable. (cusp)	11. $\int_{-1}^{1} f'(x)dx$ $f(1) - f(-1)$ $2 - (-2)$ $\boxed{4}$	12. $\int_{1}^{3} f'(x)dx$ $f(3) - f(1)$ $-2 - 2$ $\boxed{-4}$

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Question 5

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

(a) $f'(x) = 12x^2 + 2ax + b$

$f''(x) = 24x + 2a$

• point of infl. at $x = -2$

$f''(-2) = 24(-2) + 2a = 0$

$-48 + 2a = 0$

$-48 = -2a$

$\boxed{a = 24}$

• local min at $x = -1$

$f'(-1) = 12(-1)^2 + 2a(-1) + b = 0$

$12(-1)^2 + 2(24)(-1) + b = 0$

$12 - 48 + b = 0$

$\boxed{b = 36}$

Plug in $a = 24$ & $b = 36$

(b) $\int_0^1 (4x^3 + 24x^2 + 36x + k)\,dx$

$= \frac{4x^4}{4} + \frac{24x^3}{3} + \frac{36x^2}{2} + \frac{kx}{1}\Big]_0^1$

$= x^4 + 8x^3 + 18x^2 + kx\Big]_0^1$

$= \left[(1)^4 + 8(1)^3 + 18(1)^2 + k(1)\right] - \left[(0)^4 + 8(0)^3 + 18(0)^2 + k(0)\right]$

$= 1 + 8 + 18 + k - 0 - 0 - 0 - 0$

$= 27 + k$

$27 + k = 32$

$\boxed{k = 5}$

Warm-up on Day 4

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Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(a) $\int_0^{1.5} 3f'(x)+4\,dx = 3f(x)+4x\Big]_0^{1.5} = [3f(1.5)+4(1.5)] - [3f(0)+4(0)]$

$= 3(-1)+6-3(-7)-0$

$= -3+6+21-0 = \boxed{24}$

(b) point → $f(1) = -4$ $(1,-4)$

slope → $f'(1) = 5$

$\boxed{y+4 = 5(x-1)}$

$f(1.2) \approx$ $y+4 = 5(1.2-1)$

$y+4 = 5(.2)$

$y \approx \boxed{-3}$

Since $f''(x) > 0$, $f(x)$ is concave up so this approx is less than the actual value.

(c) Mean Value Theorem

- Cont & diff on $0 < c < 0.5$ ✓

- $f''(c) = \dfrac{f'(a) - f'(b)}{a-b}$

↓

$r = \dfrac{f'(0) - f'(0.5)}{0 - 0.5}$

$r = \dfrac{0-3}{-1/2}$

$\boxed{r = 6}$