AP Calculus
Unit 6 – Basic Integration & Applications

Day 2 Notes: Riemann Sums

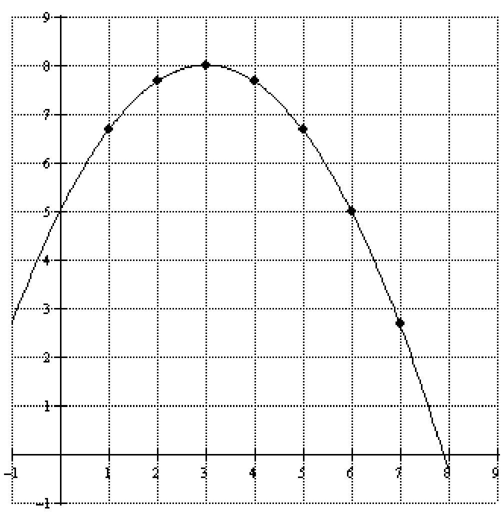
In calculus, the result of $\int f(x)dx$ is a function that represents the anti-derivative of the function $f(x)$. This is also sometimes referred to as an INDEFINITE INTEGRAL.

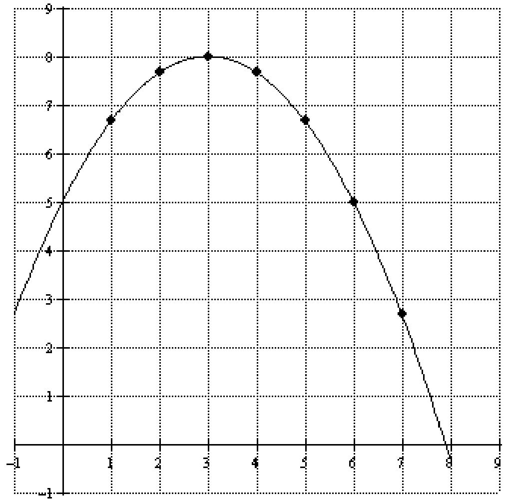
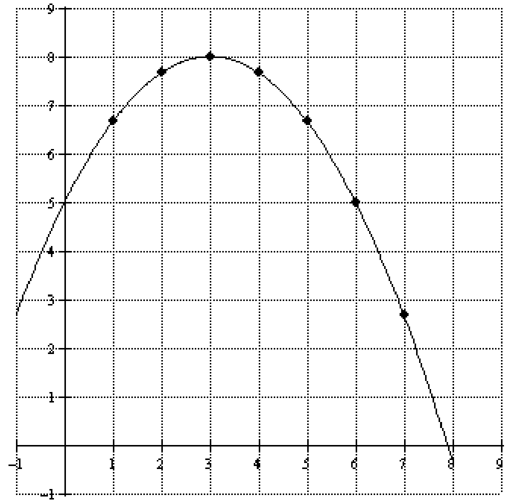
The result of $\int_a^b f(x)dx$ is a value that represents the area of the region bounded by the curve of $f(x)$ and the x – axis on the interval $a \leq x \leq b$.

Calculating Riemann sums is a way to ***estimate*** the area under a curve, the value of $\int_a^b f(x)dx$, for a graphed function on a particular interval. In this activity, you will learn to calculate four types of Riemann sums: Left Hand, Right Hand, Midpoint, and Trapezoidal Sums.

Approximation #1 – Left Hand Riemann Sum with intervals of length 2 units

Let's consider for a moment the function $f(x) = -\frac{1}{3}x^2 + 2x + 5$. This function is graphed below. On the interval [1, 7] subdivide the area bounded by the graph of the function and the x-axis into rectangles of length 2 units. Place the upper left hand vertex of the rectangle on the curve each time. Then, calculate the area of each rectangle and sum the areas to *approximate* the area of the region under the curve bounded by $f(x) = -\frac{1}{3}x^2 + 2x + 5$, $x = 1$, $x = 7$, and the x-axis.

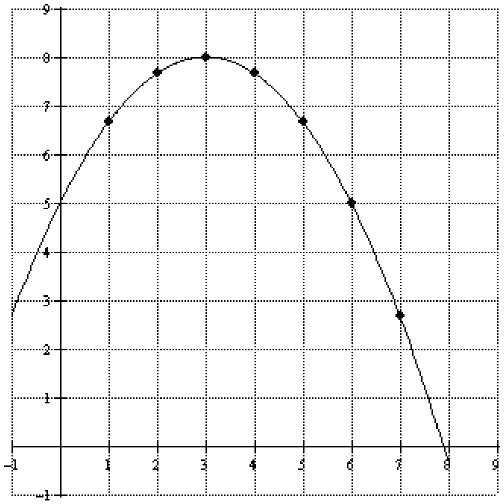


Approximation #2 – Left Hand Riemann Sum with intervals of length 1 unit	*Approximation #3 – Left Hand Riemann Sum with intervals of length ½ unit*
On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length1 unit. 	On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length ½ unit. 

Now, we are going to change things up a little bit. On the next three approximations, we are going to do a right hand sum. That is, draw your rectangles so that the upper right hand vertex of the rectangle is on the curve of the function.

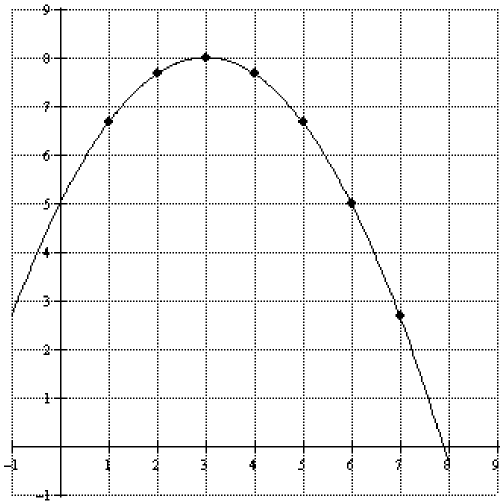
Approximation #4 – Right Hand Riemann Sum with intervals of length 2 units

On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length 2 units.



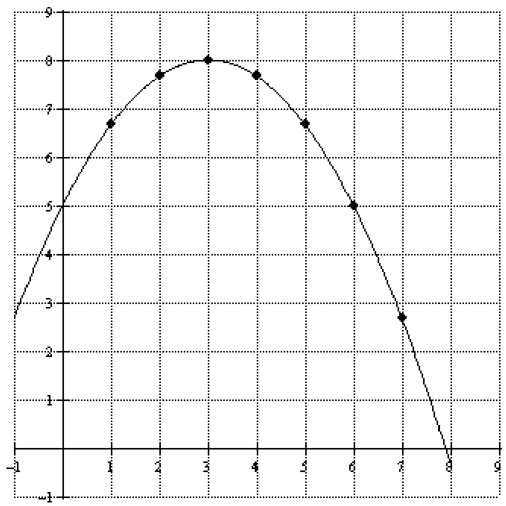
Approximation #5 – Right Hand Riemann Sum with intervals of length 1 unit

On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length 1 unit.



Approximation #6 – Right Hand Riemann Sum with intervals of length ½ unit

On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length ½ unit .



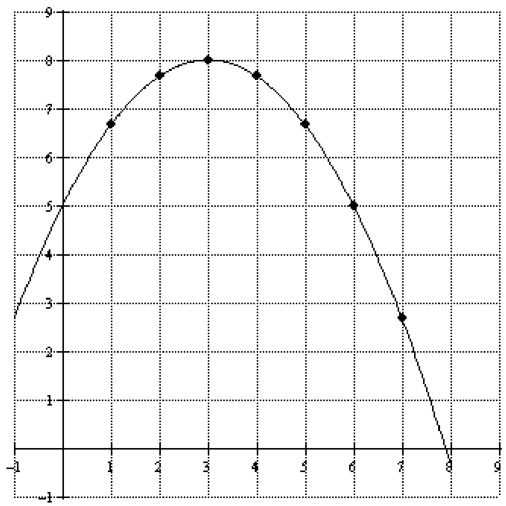
Before we continue, what inference can you make about the approximations as the lengths of the rectangles decreases?

Graphically, why do you suppose this is so?

On the next two approximations, we are going to do a midpoint sum. That is, draw your rectangles so that the midpoint of the upper left and right hand vertices of the rectangle is on the curve of the function.

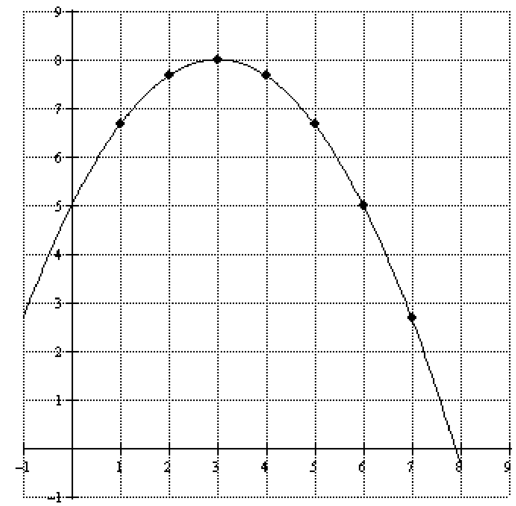
Approximation #7 – Midpoint Riemann Sum with intervals of length 2 units

On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length 2 units.



Approximation #8 – Midpoint Riemann Sum with intervals of length 1 units

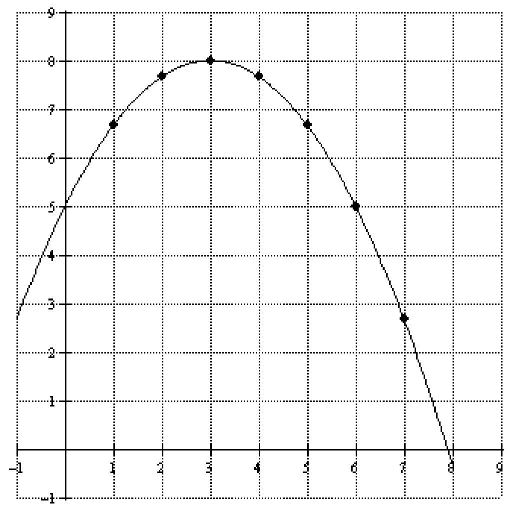
On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into rectangles of length 1 unit.



On the next two approximations, we are going to draw trapezoids between the curve and the x – axis to approximate the area. Remember that the area of a trapezoid is found using the formula $A = \frac{1}{2}h(b_1 + b_2)$, where b_1 and b_2 are the parallel sides, and h is the distance between those parallel sides.

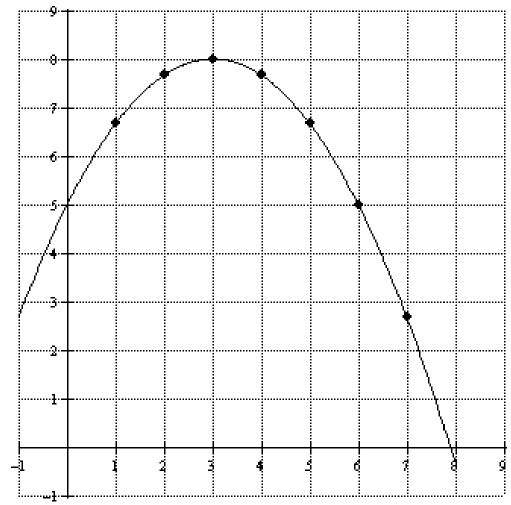
Approximation #9 – Trapezoidal Riemann Sum with intervals of length 2 units

On the interval [1, 7] subdivide the area bounded by the graph of the function and the x-axis into trapezoids of height 2 units.



Approximation #10 – Trapezoidal Riemann Sum with intervals of length 1 unit

On the interval [1, 7] subdivide the area bounded by the graph of the function and the *x*-axis into trapezoids of height 1 unit.



Type of Sum	**Numerical Approximation**	**Do you think this is an over or under approximation of the area?**
Left Hand Riemann Sum Intervals of 2 units		
Left Hand Riemann Sum Intervals of 1 unit		
Left Hand Riemann Sum Intervals of ½ unit		
Right Hand Riemann Sum Intervals of 2 units		
Right Hand Riemann Sum Intervals of 1 unit		
Right Hand Riemann Sum Intervals of ½ unit		
Midpoint Riemann Sum Intervals of 2 units		
Midpoint Riemann Sum Intervals of 1 unit		
Trapezoidal Riemann Sum Intervals of 2 units		
Trapezoidal Riemann Sum Intervals of 1 unit		

At this point, place a star next to the approximation that you feel is the most accurate to the actual area.

In our next lesson, we will learn how to find the EXACT area of a region between the graph of a function and the x – axis.

In the space below, we will come back to this to find the exact area once we complete the next lesson in order to see which approximation to the left is the most accurate.

Given the table of values below, approximate each definite integral by finding the indicated Riemann Sum.

a. Approximate $\int_0^{25} f(x)dx$ using a midpoint sum and three subintervals.

x	0	4	7	12	15	20	25
$f(x)$	15	6	–5	–10	–2	8	20

b. Approximate $\int_0^{15} f(x)dx$ using a left hand sum and four subintervals.

x	0	4	7	12	15	20	25
$f(x)$	15	6	–5	–10	–2	8	20

c. Approximate $\int_4^{20} f(x)dx$ using a right hand sum and four subintervals.

x	0	4	7	12	15	20	25
$f(x)$	15	6	–5	–10	–2	8	20

d. Approximate $\int_0^{25} f(x)dx$ using a trapezoidal sum and three subintervals

x	0	4	7	12	15	20	25
$f(x)$	15	6	–5	–10	–2	8	20

AP Calculus AB **Name: ______________________________**

Unit 6 – Day 2 – Assignment

Given below is a table of function values of $h(x)$. Approximate each of the following definite integrals using the indicated Riemann or Trapezoidal sum, using the indicated subintervals of equal length.

x	–3	–1	1	3	5	7	9
$h(x)$	5	2	–3	–7	–2	6	11

1. $\int_{-3}^{1} h(x)dx$ using two subintervals and a Left Hand Riemann sum.

2. $\int_{-3}^{9} h(x)dx$ using three subintervals and a Right Hand Riemann sum.

3. $\int_{-3}^{9} h(x)dx$ using three subintervals and a Midpoint Riemann sum.

4. $\int_{-3}^{3} h(x)dx$ using three subintervals and a Trapezoidal sum.

5. $\int_{-3}^{9} h(x)dx$ using six subintervals and a Trapezoidal sum.

6. Approximate $\int_0^{\pi}(2x\sin x)dx$ using four subintervals of equal length and a Right Hand Riemann sum.

7. Approximate $\int_{-2}^{10}\left(e^2x^2\right)dx$ using four subintervals of equal length and a Trapezoidal sum.

8. Given the table to the right, approximate $\int_{-2}^{9}P(x)dx$ using three subintervals and a Midpoint Riemann sum.

x	-2	0	1	3	5	8	9
$P(x)$	5	8	2	-4	-1	2	5

9. Given the table to the right, approximate $\int_{-2}^{9}P(x)dx$ using six subintervals and a Trapezoidal sum.

x	-2	0	1	3	5	8	9
$P(x)$	5	8	2	-4	-1	2	5