AP Calculus
Unit 6 – Basic Integration & Applications

Day 1 Notes: Finding Anti-Derivatives of Polynomial-Type Functions

$4x^2$

If you had to explain to someone how to find the derivative of a polynomial-type function, what would you say?

1. Multiply the exponent of each term by the coefficient of that term.
2. Decrease the exponent of each term by 1.

To find the anti-derivative, you would do the opposite of each one of those operations and in the reverse order. Therefore, to find the anti-derivative of a polynomial-type function....

1. Increase the exponent of each term by 1.
2. Divide the coefficient of each term by the new exponent.

The **anti-derivative (indefinite integral)** of a function, $f(x)$, is denoted by the notation $\int f(x)dx$. So when finding the anti-derivative of a function, you are finding the function of which $f(x)$ is the first derivative. This will enable us, if given f' or f'' to be able to find f. However, if $\int f'(x)dx = f(x)$, what problem do you foresee?

The problem is that when $\int f'(x)\,dx$ is found, there is no way of knowing if there was a constant term of $f(x)$ and if there was, what it could be.

$$\int f'(x)\,dx = f(x) + C$$

$$\frac{ax^0}{0} \rightarrow \int \frac{a}{x} = a\ln x$$

Find each of the following anti-derivatives.

$\int(3x^2+2x+3)dx$ $= \frac{3x^3}{3} + \frac{2x^2}{2} + \frac{3x^1}{1} + C$ $= \boxed{x^3 + x^2 + 3x + C}$	$\int\left(\frac{x^3+2x-4}{x}\right)dx$ $= \int x^2 + 2 - 4x^{-1}\,dx$ $= \frac{x^3}{3} + \frac{2x}{1} - \frac{4x^0}{0} + C$ $= \boxed{\frac{1}{3}x^3 + 2x - 4\ln x + C}$
$\int(x+2)(2x-3)dx$ $= \int 2x^2 - 3x + 4x - 6\,dx$ $= \int 2x^2 + x - 6\,dx$ $= \frac{2x^3}{3} + \frac{x^2}{2} - \frac{6x}{1} + C$ $= \boxed{\frac{2}{3}x^3 + \frac{1}{2}x^2 - 6x + C}$	$\int \frac{2}{\sqrt{x}}dx = \int 2x^{-1/2}dx$ $= \frac{2x^{1/2}}{1/2} + C$ $= 4x^{1/2} + C$ $= \boxed{4\sqrt{x} + C}$

We learned that $\frac{d}{dx}[\sin x] = \cos x$ and $\frac{d}{dx}[\cos x] = -\sin x$. Similarly, write what the anti-derivatives of sine and cosine are.

$\int \cos x\,dx =$ $\sin x + C$ $\qquad$ $\int \sin x\,dx =$ $-\cos x + C$

Find each of the following anti-derivatives.

$\int(2\sin x + \cos x)dx$ $= \boxed{-2\cos x + \sin x + C}$	$\int(t^2 - \sin t)dt$ $= \frac{t^3}{3} - (-\cos t) + C$ $= \boxed{\frac{1}{3}t^3 + \cos t + C}$

$\int (4x - 3\cos x)dx$ $= \frac{4x^2}{2} - 3(\sin x) + C$ $= \boxed{2x^2 - 3\sin x + C}$	$\int (\sqrt{x} + \sin x)dx$ $= \int x^{1/2} + \sin x \, dx$ $= \frac{x^{3/2}}{3/2} - \cos x + C$ $= \boxed{\frac{2}{3}x^{3/2} - \cos x + C}$

Use the given information about f' and f'' to find $f(x)$.

1. $f''(x) = 2$ $\quad f'(2) = 5$ $\quad f(2) = 10$	2. $f''(x) = x^{-3/2}$ $\quad f'(4) = 2$ $\quad f(0) = 0$
$f'(x) = \int f''(x) = \int 2\,dx$ $= \frac{2x}{1} + C$ $f'(x) = 2x + C$ $f'(2) = 2(2) + C = 5$ $C = 1$ $f'(x) = 2x + 1$ $f(x) = \int f'(x) = \int (2x+1)dx$ $= \frac{2x^2}{2} + \frac{1x}{1} + C$ $f(x) = x^2 + x + C$ $f(2) = (2)^2 + 2 + C = 10$ $C = 4$ $\boxed{f(x) = x^2 + x + 4}$	$f'(x) = \int f''(x) = \int x^{-3/2}dx$ $= \frac{x^{-1/2}}{-1/2} + C$ $f'(x) = -2x^{-1/2} + C$ $f'(4) = -2(4)^{-1/2} + C = 2$ $-\frac{2}{\sqrt{4}} + C = 2$ $C = 3$ $f'(x) = -2x^{-1/2} + 3$ $f(x) = \int f'(x) = \int -2x^{-1/2} + 3$ $= \frac{-2x^{1/2}}{1/2} + \frac{3x}{1} + C$ $f(x) = -4x^{1/2} + 3x + C$ $f(0) = \cancel{-4(0)^{1/2}} + 3\cancel{(0)} + C = 0$ $C = 0$ $\boxed{f(x) = -4\sqrt{x} + 3x}$

An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by the differential equation

$$\frac{dh}{dt} = 1.5t + 5,$$

derivative

where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted, at $t = 0$.

$h(0) = 12$

a. Find the value of the differential equation above when $t = 3$. Using correct units of measure, explain what this value represents in the context of this problem.

$\frac{dh}{dt} = 1.5(3) + 5 = \boxed{9.5\text{ cm/yr}}$

Three years after planting, the height of the shrub is increasing at a rate of 9.5 cm per year.

b. Find an equation for $h(t)$, the height of the shrubs at any year t. Then, determine how tall the shrubs are when they are sold.

$h(t) = \int \frac{dh}{dt} = \int 1.5t + 5\, dt$

$= \frac{1.5t^2}{2} + \frac{5t}{1} + C$

$h(t) = 0.75t^2 + 5t + C$

$h(0) = 0.\cancel{75(0)}^2 + 5\cancel{(0)} + C = 12$

$C = 12$

$\boxed{h(t) = 0.75t^2 + 5t + 12}$

$h(6) = 0.75(6)^2 + 5(6) + 12$

$\boxed{h(6) = 69\text{ cm}}$

A particle moves along the x – axis at a velocity of $v(t) = \frac{1}{\sqrt{t}}$, for $t > 0$. At time $t = 1$, its position is 4.

$v(t) = t^{-1/2}$ $p(1) = 4$

a. What is the acceleration of the particle particle when $t = 9$?

$a(t) = v'(t) = -\frac{1}{2}t^{-3/2}$

$a(t) = \frac{-1}{2\sqrt{t^3}}$

$a(9) = \frac{-1}{2\sqrt{9^3}}$

$= \frac{-1}{2\sqrt{729}} = \frac{-1}{2(27)} = \boxed{\frac{-1}{54}}$

b. What is the position of the when $t = 9$?

$p(t) = \int v(t) = \int t^{-1/2}$

$= \frac{t^{1/2}}{1/2} + C$

$p(t) = 2\sqrt{t} + C$

$p(1) = 2\sqrt{1} + C = 4$

$C = 2$

$p(t) = 2\sqrt{t} + 2 \rightarrow p(9) = 2\sqrt{9} + 2$

$\boxed{p(9) = 8}$