

AP Calculus  
Unit 6 – Basic Integration & Applications

## Day 1 Notes: Finding Anti-Derivatives of Polynomial-Type Functions

If you had to explain to someone how to find the derivative of a polynomial-type function, what would you say?

To find the anti-derivative, you would do the opposite of each one of those operations and in the reverse order. Therefore, to find the anti-derivative of a polynomial-type function....

The **anti-derivative (indefinite integral)** of a function,  $f(x)$ , is denoted by the notation  $\int f(x)dx$

. So when finding the anti-derivative of a function, you are finding the function of which  $f(x)$  is the first derivative. This will enable us, if given  $f'$  or  $f''$  to be able to find  $f$ . However, if

$\int f'(x)dx = f(x)$ , what problem do you foresee?

**Find each of the following anti-derivatives.**

$\int (3x^2 + 2x + 3) dx$	$\int \left( \frac{x^3 + 2x - 4}{x} \right) dx$
$\int (x + 2)(2x - 3) dx$	$\int \frac{2}{\sqrt{x}} dx$

We learned that  $\frac{d}{dx}[\sin x] = \cos x$  and  $\frac{d}{dx}[\cos x] = -\sin x$ . Similarly, write what the anti-derivatives of sine and cosine are.

$$\int \cos x dx = \underline{\hspace{2cm}}$$

$$\int \sin x dx = \underline{\hspace{2cm}}$$

**Find each of the following anti-derivatives.**

$\int (2 \sin x + \cos x) dx$	$\int (t^2 - \sin t) dt$
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$\int (4x - 3 \cos x) dx$	$\int (\sqrt{x} + \sin x) dx$
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**Use the given information about  $f'$  and  $f''$  to find  $f(x)$ .**

1. $f''(x) = 2$ $f'(2) = 5$ $f(2) = 10$	2. $f''(x) = x^{-3/2}$ $f'(4) = 2$ $f(0) = 0$

**An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by the differential equation**

$$\frac{dh}{dt} = 1.5t + 5,$$

**where  $t$  is the time in years and  $h$  is the height in centimeters. The seedlings are 12 centimeters tall when planted, at  $t = 0$ .**

a. Find the value of the differential equation above when  $t = 3$ . Using correct units of measure, explain what this value represents in the context of this problem.

b. Find an equation for  $h(t)$ , the height of the shrubs at any year  $t$ . Then, determine how tall the shrubs are when they are sold.

**A particle moves along the  $x$  – axis at a velocity of  $v(t) = \frac{1}{\sqrt{t}}$ , for  $t > 0$ . At time  $t = 1$ , its position is 4.**

a. What is the acceleration of the particle when  $t = 9$ ?

b. What is the position of the particle when  $t = 9$ ?

**AP Calculus AB**  
**Unit 6 – Day 1 – Assignment**

Name: \_\_\_\_\_

For problems 1 – 12, find the indefinite integrals below.

1. $\int (\sqrt[3]{x} + 3) dx$	2. $\int (2x - 3x^2) dx$
3. $\int x^2(2x^2 + 3x) dx$	4. $\int (x^{3/2} + 2x + 1) dx$
5. $\int \left( \sqrt{x} + \frac{1}{2\sqrt{x}} \right) dx$	6. $\int \frac{3x^2 - 2x + 3}{x^3} dx$
7. $\int y^3 \sqrt{y} dy$	8. $\int \frac{1}{w\sqrt{w}} dw$

9.  $\int \frac{x^3+3}{\sqrt{x}} dx$

10.  $\int (x+3)(x-3)^2 dx$

11.  $\int (\theta^2 + \cos \theta) d\theta$

12.  $\int (\sqrt{x} - \sin x + 2) dx$

For problems 13 and 14, find the indicated function based on the given information.

13. If  $f'(x) = 2x - \sin x$  and  $f(0) = 4$ , find  $f(x)$ .

14. If  $f''(x) = x^2$ ,  $f'(0) = 6$ , and  $f(0) = 3$ , find  $f(x)$ .