AP Calculus AB
Unit 6 – Day 1 – Assignment

Name: Answer Key*

For problems 1 – 12, find the indefinite integrals below.

1. $\int \left(\sqrt[3]{x}+3\right)dx = \int x^{1/3}+3\,dx$

$= \frac{x^{4/3}}{4/3} + \frac{3x}{1} + C$

$\boxed{\frac{3}{4}x^{4/3}+3x+C}$

2. $\int \left(2x-3x^2\right)dx$

$= \frac{2x^2}{2} - \frac{3x^3}{3} + C$

$= \boxed{x^2 - x^3 + C}$

3. $\int x^2\left(2x^2+3x\right)dx = \int 2x^4+3x^3\,dx$

$= \frac{2x^5}{5} + \frac{3x^4}{4} + C$

$= \boxed{\frac{2}{5}x^5+\frac{3}{4}x^4+C}$

4. $\int \left(x^{3/2}+2x+1\right)dx$

$= \frac{x^{5/2}}{\frac{5}{2}} + \frac{2x^2}{2} + \frac{1x}{1} + C$

$= \boxed{\frac{2}{5}x^{5/2}+x^2+x+C}$

5. $\int \left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)dx = \int x^{1/2}+\frac{1}{2}x^{-1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + \frac{\frac{1}{2}x^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{3}x^{3/2}+x^{1/2}+C}$

6. $\int \frac{3x^2-2x+3}{x^3}dx = \int 3x^{-1}-2x^{-2}+3x^{-3}\,dx$

$\frac{3x^0}{0} - \frac{2x^{-1}}{-1} + \frac{3x^{-2}}{-2} + C$

$\boxed{3\ln x + \frac{2}{x} - \frac{3}{2x^2} + C}$

7. $\int y^3\sqrt{y}\,dy = \int y^3 y^{1/2}\,dy = \int y^{7/2}\,dy$

$= \frac{y^{9/2}}{9/2} + C = \boxed{\frac{2}{9}y^{9/2}+C}$

8. $\int \frac{1}{w\sqrt{w}}dw = \int w^{-1}w^{-1/2}dw = \int w^{-3/2}\,dw$

$= \frac{w^{-1/2}}{-1/2} + C = -2w^{-1/2}+C$

$= \boxed{\frac{-2}{\sqrt{w}}+C}$

9. $\int \frac{x^3+3}{\sqrt{x}}\,dx = \int \frac{x^3+3}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{-1/2}\,dx$

$= \frac{x^{7/2}}{7/2} + \frac{3x^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{7}x^{7/2} + 6x^{1/2} + C}$

10. $\int (x+3)(x-3)^2\,dx = \int (x+3)(x^2-6x+9)\,dx$

$= \int (x^3 - 6x^2 + 9x + 3x^2 - 18x + 27)\,dx$

$= \int (x^3 - 3x^2 - 9x + 27)\,dx$

$= \frac{x^4}{4} - \frac{3x^3}{3} - \frac{9x^2}{2} + \frac{27x}{1} + C$

$= \boxed{\frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 + 27x + C}$

11. $\int (\theta^2 + \cos\theta)\,d\theta$

$= \frac{\theta^3}{3} + \sin\theta + C$

$= \boxed{\frac{1}{3}\theta^3 + \sin\theta + C}$

12. $\int (\sqrt{x} - \sin x + 2)\,dx = \int (x^{1/2} - \sin x + 2)\,dx$

$= \frac{x^{3/2}}{3/2} + \cos x + \frac{2x}{1} + C$

$= \boxed{\frac{2}{3}x^{3/2} + \cos x + 2x + C}$

For problems 13 and 14, find the indicated function based on the given information.

13. If $f'(x) = 2x - \sin x$ and $f(0) = 4$, find $f(x)$.

$\int f'(x)\,dx = \int 2x - \sin x\,dx$

$f(x) = \frac{2x^2}{2} + \cos x + C$

$f(0) = \cancel{(0)^2} + \cos 0 + C = 4$

$1 + C = 4$

$C = 3$

$\boxed{f(x) = x^2 + \cos x + 3}$

14. If $f''(x) = x^2$, $f'(0) = 6$, and $f(0) = 3$, find

$\int_{f(x)} f''(x)\,dx = \int x^2\,dx = \frac{x^3}{3} + C$

$f'(x) = \frac{1}{3}x^3 + C$

$f'(0) = \cancel{\frac{1}{3}(0)^3} + C = 6$

$C = 6$

$f'(x) = \frac{1}{3}x^3 + 6$

$\int f'(x)\,dx = \int \frac{1}{3}x^3 + 6\,dx$

$= \frac{\frac{1}{3}x^4}{4} + \frac{6x}{1} + C$

$f(x) = \frac{1}{12}x^4 + 6x + C$

$f(0) = \frac{1}{12}\cancel{(0)^4} + 6\cancel{(0)} + C = 3$

$C = 3$

$\boxed{f(x) = \frac{1}{12}x^4 + 6x + 3}$