(calc.)

Administrators at a hospital believe that the number of beds in use is given by the function
$B(t) = 20\sin(t/10) + 50$, where t is measured in days.
For $12 \le t \le 20$, what is the minimum number of beds in use?

A.

Extreme val. Thm
- cont ✓

$B'(t) = 20\cos(\frac{t}{10})(\frac{1}{10})$

$B'(t) = \underbrace{2\cos(t/10)}_{y_1} = \underbrace{0}_{y_2}$

$t = 15.708$

$B(12) = 68.641$
$B(20) = 68.186$ ← min
$B(15.708) = 70$

$\boxed{68.186}$

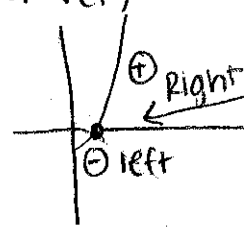
(calc Active)

If $p(t) = e^{2t} - 6t$ represents the position function of a particle, when does the particle change directions?

I.

$v(t) = 2e^{2t} - 6 = 0$
$2(e^{2t} - 3) = 0$
$e^{2t} = 3$
$\ln 3 = 2t$
$\frac{\ln 3}{2} = t$
$\boxed{t = 0.549}$

Graph of v(t)



(no calc.)

If f(x) is continuous on [a, b] and differentiable on (a, b) and f(a) = f(b), what theorem can you use to find the value of c on (a, b) such that $\boxed{f'(c) = 0}$?

Rolle's Theorem

N.

(NO CALC)

Find:

$$\lim_{x\to 1} \frac{x - 1}{\ln x - \sin(\pi x)}$$

$$\lim_{x\to 1} \frac{1}{\frac{1}{x} - \pi\cos(\pi x)} = \frac{1}{\frac{1}{1} - \pi\cos(\pi)}$$

R.

$$= \frac{1}{1 - \pi(-1)}$$

$$= \boxed{\frac{1}{1+\pi}}$$

(NO calc)

✱ If the velocity function is given by $v(t) = \sin(\pi x)$, what is the acceleration of this particle at $t = 2$?

$v'(t) = a(t) = \pi\cos(\pi x)$

$a(2) = \pi\cos(2\pi)$

X.

$= \pi(1) = \boxed{\pi}$

(NO calc)

Find: $\lim_{x \to \infty} \dfrac{2x^2 + 3x}{x^3 + x + 1}$

$\lim_{x \to \infty} \dfrac{4x+3}{3x^2+1} = \dfrac{\infty}{\infty}$

H.

$\lim_{x \to \infty} \dfrac{4}{6x} = \dfrac{4}{\infty} = \boxed{0}$

(NO calc)

x	0	2	4	6	8	10	12	14	16
$f(x)$	1	5	8	10	11	10	8	5	1

Find the average rate of change of f(x) on the interval [4, 14].

$= \dfrac{f(4) - f(14)}{4-14}$

S.

$= \dfrac{8-5}{4-14} = \boxed{\dfrac{3}{-10}}$

(NO calc.)

For what value of c is the instantaneous rate of change for the function $f(x) = 2\sqrt{x}$ equal to the average rate of change on the interval $1 \le x \le 4$?

$f(x) = 2x^{1/2}$ $f'(x) = x^{-1/2}$

E.

$= \dfrac{1}{\sqrt{x}}$

$f(1) = 2\sqrt{1} = 2$

$f(4) = 2\sqrt{4} = 4$

$\dfrac{1}{\sqrt{c}} = \dfrac{f(1) - f(4)}{1-4}$

$\dfrac{1}{\sqrt{c}} \times \dfrac{2-4}{1-4} \rightarrow$

$-3 = -2\sqrt{c}$

$\dfrac{3}{2} = \sqrt{c}$ $\boxed{c = \dfrac{9}{4}}$

(No calc)

$p'(t) =$ $v(t) = (t - 1)(t - 3)^2$ is velocity of a particle where t is measured in minutes and v(t) is measured in inches per minute. At what interval is the particle moving to the right? $v(t)$ $+$

$(t-1)(t-3)^2 = 0$

M. $t=1 \quad t=3$

.5   2   4

0 (−)(+) 1 (+)(+) 3 (+)(+)

−   +   +

Right

$\boxed{(1,3) \cup (3,\infty)}$

(No calc)

If $f(x) = x^2 - 5x$ on the interval [0, 5], find the value of c such that $f'(c) = 0$.

$f'(x) = 2x - 5$

$f'(c) = 2c - 5 = 0$

T. $\boxed{c = 5/2}$

(No calc)

x	0	2	3	4	6
$g(x)$	−3	1	5	2	1

g(x) is a differentiable function on the interval [0, 6].
On what interval is there guaranteed to be a value of c such that $g(c) = -1$?

BB. Intermediate Value Thm

$\boxed{(0,2)}$

$g(0) = -3$
$g(2) = 1$ > −1 is between −3 & 1

(Calc)

The velocity of a particle is given by $v(t) = -2 + (t^2 + 3)^4$. At t = 2.5, what direction is the particle moving?

$v(2.5) = -2 + (2.5^2 + 3)^4$

$= 7318.941$

B. Since $v(2.5) > 0$, particle moving to $\boxed{\text{Right}}$

(No calc.)

If f(x) is continuous on [a, b], what theorem can you use to find the absolute extrema of a function?

$\boxed{\text{Extreme Value Theorem}}$

J.

(Calc.)

If $p(t) = e^{2t} - 8t$ represents the position function of a particle, what is the total distance the particle travels on [0.5, 1.5]?

$v(t) = 2e^{2t} - 8$

$2e^{2t} - 8 = 0$

$2e^{2t} = 8$

$e^{2t} = 4$

$\ln 4 = 2t$

$t = 0.693$ ↑ changes directions

CC.

Total Dist =

$|p(0.5) - p(.693)| + |p(.693) - p(1.5)|$

$|-1.282 - (-1.545)| + |-1.545 - 8.086|$

$.263 + 9.631 = \boxed{9.894}$

(calc.)

$v(t) = (t + 1)(t + 3)^2$ is velocity of a particle where t is measured in minutes and v(t) is measured in inches per minute. Describe the speed of the particle at t = 2 minutes.

$v(2) = (2+1)(2+3)^2 = 75$

$v'(2) = a(2) = 55.000$

L.

math 8

$\boxed{\text{Speed is increasing}}$

(No calc)

Find the point on the graph of $f(x) = \sqrt{-x + 10}$ so that the point (2, 0) is closest to the graph.

minimize distance

$d = \sqrt{(x-2)^2 + (y-0)^2}$

O.

$d = \sqrt{(x-2)^2 + (\sqrt{-x+10})^2}$

$d = \sqrt{x^2 - 4x + 4 - x + 10}$

$d = \sqrt{x^2 - 5x + 14} = (x^2 - 5x + 14)^{1/2}$

$d'(x) = \frac{1}{2}(x^2 - 5x + 14)^{-1/2}(2x - 5)$

$d'(x) = \frac{2x - 5}{2\sqrt{x^2 - 5x + 14}} = 0$

$2x - 5 = 0$

$x = 5/2$

0 (−) 5/2 (+) 3

min

$x = \frac{5}{2}$

$y = \sqrt{-\frac{5}{2} + 10} = \sqrt{\frac{15}{2}}$

$\boxed{\left(\frac{5}{2}, \sqrt{\frac{15}{2}}\right)}$

(No calc)

If f(x) is continuous on [a, b] and differentiable on (a, b), then what theorem would you use to find $f'(c) = \frac{f(a) - f(b)}{a - b}$?

Mean Value Thm

AA.

(calc. Active)

A particle's position is given by $p(t) = e^t \cos t$, where p(t) is measured in centimeters and t is measured in seconds. What is the instantaneous acceleartion ~~on [1, 3]~~? at t=1.5?

U. $v(t) = (e^t)(\cos t) + (e^t)(-\sin t)$

$a(t) = \cancel{(e^t)(\cos t)} + (e^t)(-\sin t) + (e^t)(-\sin t) + \cancel{(e^t)(\cos t)}$

$a(t) = -2e^t \sin t$

$a(1.5) = -2e^{1.5}\sin(1.5) = \boxed{-8.941}$

(calc).

Extreme Value Thm f(x) cont on [-5, 6] ✓

If $f(x) = (x + 2)^{2/3}$ on [-5, 6], what is the absolute ~~minimum~~ max of f(x)? ~~(Calc: Inactive)~~

$f'(x) = \frac{2}{3}(x+2)^{-1/3}$

$= \frac{2}{3\sqrt[3]{x+2}}$ ← undefined at $x=-2$

F.

endpts $f(-5) = 2.080$

$f(6) = 4$ ← $\boxed{max = 4}$

f'(x) undef. $f(-2) = 0$

(No calc)

$f'(c) = \frac{f(a) - f(b)}{a - b}$

Apply the Mean Value Theorem to find the value(s) of c guaranteed for $f(x) = x^3 - x^2 - 2x$ on [-1, 1]. $f'(x) = 3x^2 - 2x - 2$

V. $3c^2 - 2c - 2 = \frac{f(-1) - f(1)}{-1 - 1}$

$3c^2 - 2c - 2 = \frac{0 - (-2)}{-2} = -1$

$3c^2 - 2c - 2 = -1$

$3c^2 - 2c - 1 = 0$

$(3c+1)(c-1)$

$\boxed{c = -1/3}$ $\cancel{c=1}$ ← endpt. on interval

(NO calc)

x	0	2	3	4	6
$g(x)$	–3	1	5	2	1

g(x) is a differentiable function on the interval [0, 6].
On what interval is there guaranteed to be a value of c such that g'(c) =0?

DD.

Rolle's Thm
- cont ✓
- diff ✓

$g(2)=1$

$g(6)=1$

$\boxed{(2,6)}$

(NO calc)

A particle moves along a line so that at time t, where 0≤t≤π, its position is given by s(t) = -4cost - 2t.
What is the velocity of the particle when its acceleration is zero?

W.

$v(t) = 4\sin t - 2$

$a(t) = 4\cos t$

$4\cos t = 0$

$\cos t = 0$

$\frac{\pi}{2}$

$v(\frac{\pi}{2}) = 4\sin(\frac{\pi}{2}) - 2$

$= 4(1) - 2$

$= \boxed{2}$

(calc).

A car company introduces a new car for which the number of cars sold, S, is modeled by the function
$S(t) = 1500 - \frac{45}{t+2}$ where t is time in months.

Find the average rate of change of cars sold over the first 6 months.

C.

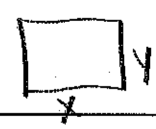
$\frac{S(0)-S(6)}{0-6} = \frac{1477.5-1494.375}{-6}$

$= \boxed{2.813}$

(no calc)

The area of a rectangle is 81 square feet. What dimensions of the rectangle would give the smallest perimeter?

minimize perimeter



$P = 2x + 2y$

P.

$81 = xy$

$\frac{81}{x} = y$

$P = 2x + 2(\frac{81}{x})$

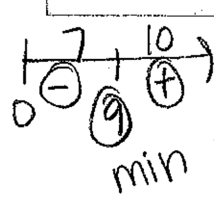
$P = 2x + 162x^{-1}$

$P'(x) = 2 - \frac{162}{x^2} = 0$

$\frac{-162}{x^2} = -2$

$162 = 2x^2$

$x^2 = 81 \quad x = \pm 9$



$\boxed{\text{9ft by 9ft}}$

(NO calc)

Find: $\lim_{x \to 0} \frac{e^x - 1}{4x}$

$\lim_{x \to 0} \frac{e^x}{4} = \frac{e^0}{4} = \boxed{\frac{1}{4}}$

Q.

(NO calc)

x	0	2	3	4	6
g(x)	−3	1	5	2	1

g(x) is a differentiable function on the interval [0, 6].
On what interval is there guaranteed to be a value of c such that g'(c) =4?

K.

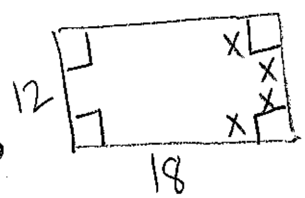
mean val. Thm
- cont ✓
- diff ✓

$\frac{g(2) - g(3)}{2-3} = \frac{1-5}{-1} = \frac{-4}{-1} = 4$ $\boxed{(2,3)}$

$Vol = (1.662)(12-2(1.662))(18-2(1.662))$

$V = (1.662)(8.676)(14.676)$

(calc) $\boxed{V = 211.621}$



maximize volume

Find the maximum volume of a box that can be made by cutting squares from the corners of an 12 inch by 18 inch rectangular sheet of cardboard and folding up the sides.

$V = (x)(12-2x)(18-2x)$

Y. $V = (12x - 2x^2)(18-2x)$

$V = 216x - 24x^2 - 36x^2 + 2x^3$

$V = 216x - 60x^2 + 2x^3$

$V'(x) = \underbrace{216 - 120x + 6x^2}_{y_1} = \underbrace{0}_{y_2}$

$x = 1.662$

0 (+) 1.662 (−) 2 max

(NO calc)

$f'(c) = \frac{f(a) - f(b)}{a-b}$

If c is the number that satisfies the conclusion of the Mean Value Theorem for f(x) = x^3 - 4x^2 on [0, 2], the c = ?

$f'(x) = 3x^2 - 8x$ $\frac{f(0) - f(2)}{0-2} = \frac{0-(-8)}{-2} = -4$

G.

$3c^2 - 8c = -4$

$3c^2 - 8c + 4 = 0$

$(c-2)(3c-2)$

endpt $\cancel{c=2}$ $c = 2/3$

$\frac{2}{3}$

(calc)

A particle's position is given by $p(t) = e^t \cos t$, where $p(t)$ is measured in centimeters and t is measured in seconds. What is the average velocity on [1, 3]?

$$\frac{p(1)-p(3)}{1-3} = \frac{1.469-(-19.885)}{-2}$$

Z.

$$= \boxed{-10.677}$$

Extreme Value Thm
- cont ✓

If $g(x) = 2x^2 - 4x$ on the interval [0, 5], what is the absolute maximum of $g(x)$? (Calc. Inactive)

$g'(x) = 4x - 4$ $\quad g(0) = 0$

$4x - 4 = 0$ $\quad g(5) = 30$

D. $\quad x = 1$ $\quad g(1) = -2$

$\boxed{30}$