AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 4 & 5 Notes: Particle Motion Problems

Average and Instantaneous Velocity

Example 1: A particle's position is given by the function $p(t) = e^t \sin t$, where $p(t)$ is measured in centimeters and t is measured in seconds. Answer the following questions.

a) What is the average velocity on the interval $t = 1$ to $t = 3$ seconds? Indicate appropriate units of measure.

b) What is the instantaneous velocity of the particle at time $t = 1.5$. Indicate appropriate units of measure.

Average and Instantaneous Acceleration

c) What is the average acceleration on the interval $t = 1$ to $t = 3$ seconds? Indicate appropriate units of measure.

d) What is the instantaneous acceleration of the particle at time $t = 1.5$.

In summary, let's correlate the concepts of position, velocity, and acceleration to what we already know about a function and its first and second derivative.

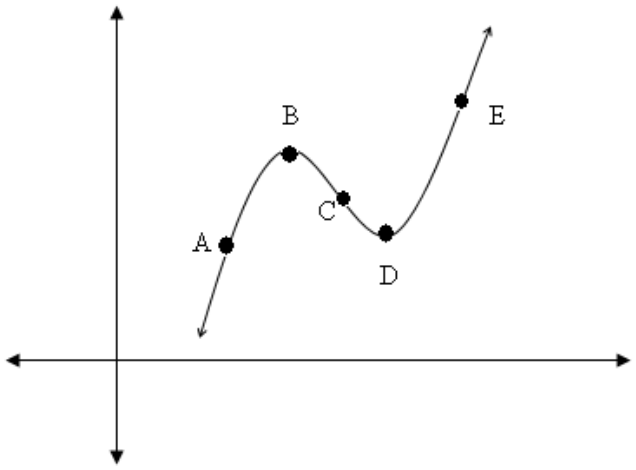
	corresponds with	
	corresponds with	
	corresponds with	

Let's summarize our relationships between position, velocity and acceleration below.

Velocity	***Position (Motion of the Particle)***
Is = 0 or is undefined	
Is > 0	
Is < 0	
Changes from positive to negative	
Changes from negative to positive	

Acceleration	***Velocity***
Is = 0 or is undefined	
Is > 0	
Is < 0	
Changes from positive to negative	
Changes from negative to positive	

The graph below represents the position, $s(t)$, of a particle which is moving along the x axis.



- At which point(s) is the velocity equal to zero? Justify your answer.

- At which point(s) does the acceleration equal zero? Justify your answer.

- On what interval(s) is the particle's velocity positive? Justify your answer.

- On what interval(s) is the particle's velocity negative? Justify your answer.

- On what interval(s) is the particle's acceleration positive? Justify your answer.

- On what interval(s) is the particle's acceleration negative? Justify your answer.

Five Commandments of Particle Motion

1.
2.
3.
4.
5.

Example 2: Suppose the velocity of a particle is given by the function $v(t) = (t+2)(t+4)^2$ for $t \geq 0$, where t is measured in minutes and $v(t)$ is measured in inches per minute. Answer the questions that follow.

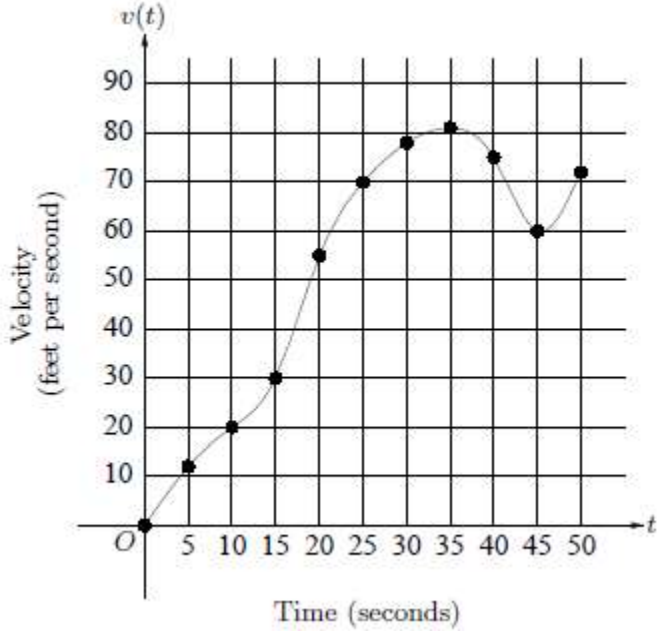
a) Find the values of $v(3)$ and $v'(3)$. Based on these values, describe the speed of the particle at $t = 3$.

b) On what interval(s) is the particle moving to the left? Right? Show your analysis and justify your answer.

AP Calculus AB **Name:** _____
Unit 5 – Days 4 & 5 – Assignment

1) 1998 AP Calculus AB #3 (Modified)

The graph of the velocity $v(t)$, in feet per second, of a car traveling on a straight road, for $0 \leq t \leq 50$ is shown below. A table of values for $v(t)$, at 5 second intervals of time, is also shown to the right of the graph.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

a. During what interval(s) of time is the acceleration of the car positive? Give a reason for your answer.

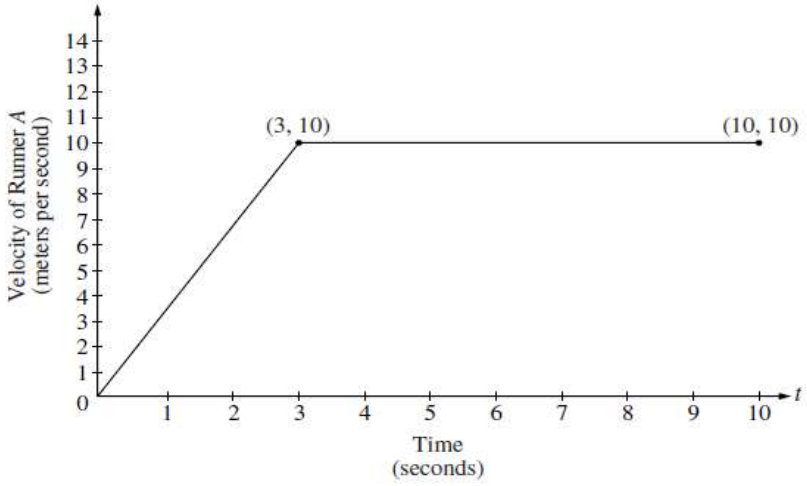
b. Find the average acceleration of the car over the interval $0 \leq t \leq 50$. Indicate units of measure.

c. Find one approximation for the acceleration of the car at $t = 40$. Show the computations you used to arrive at your answer. Indicate units of measure.

d. Is the speed of the car increasing or decreasing at $t = 40$? Give a reason for your answer.

2) 2000 AP Calculus AB #2 (Partial)

Two runners, *A* and *B*, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner *A*. The velocity, in meters per second, of Runner *B* is given by the function *v* defined by $v(t) = \frac{24t}{2t+3}$.



a. Find the velocity of Runner A and the velocity of Runner B at $t = 2$ seconds. Indicate units of measure.

b. Find the acceleration of Runner *A* and the acceleration of Runner *B* at time $t = 2$ seconds. Indicate units of measure.

3) 2002 AP Calculus AB #3 (Partial)

An object moves along the x – axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by the function $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

a. What is the acceleration of the object at time $t = 4$?

b. Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is decreasing.

Are either or both of these statements correct? For each statement, provide a reason why it is correct or not correct.

A particle moves along the x axis such that its position, for $t > 0$, is given by the function $p(t) = e^{2t} - 5t$. Use this information to complete exercises 4 – 7.

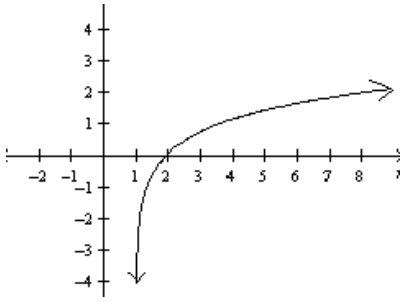
4. What are the values of $p'(2)$ and $p''(2)$? Explain what each value represents.

5. Based on the values found in part (a), what can be concluded about the speed of the particle at $t = 2$? Give a reason for your answer.

6. On what interval(s) of t is the particle moving to the left? To the right? Justify your answers.

7. Does the particle ever change directions? Justify your answer.

8. The graph of $v(t)$, the velocity of a moving particle, is given below. What conclusions can be made about the movement of the particle along the x – axis and the acceleration, $a(t)$, of the particle for $t > 0$? Give reasons for your answers.



9. If the position of a particle is defined by the function $x(t) = t^3 - 9t^2 + 24t$ for $t > 0$, is the speed of the particle increasing or decreasing when $t = 2.5$? Justify your answer.

The position of a particle is given by the function $p(t) = (2t - 3)e^{2-t}$ for $t > 0$. Answer questions 10 – 12.

10. What is the average velocity from $t = 1$ to $t = 3$?

11. Find an equation for $v(t)$, the velocity of the particle.

12. For what value(s) of t will the $v(t) = 0$?

2003 AP Calculus AB #2 (Partial)

A particle moves along the x – axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$.

13. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain why or why not.

14. Find all times in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.