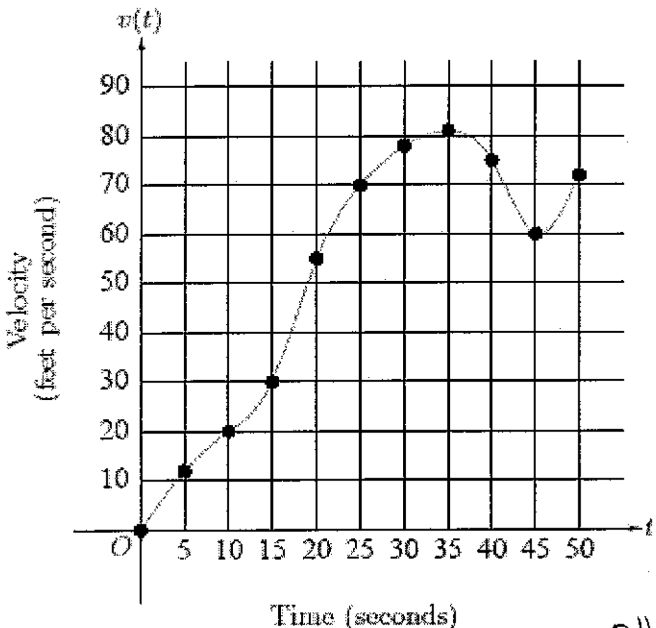
AP Calculus AB
Unit 5 – Days 4 & 5 – Assignment

Name: Answer Key*

1) 1998 AP Calculus AB #3 (Modified)

The graph of the velocity $v(t)$, in feet per second, of a car traveling on a straight road, for $0 \le t \le 50$ is shown below. A table of values for $v(t)$, at 5 second intervals of time, is also shown to the right of the graph.

$p'(t)$



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

$p''(t) > 0$

a. During what interval(s) of time is the acceleration of the car positive? Give a reason for your answer.

If $a(t) > 0$, then $v(t)$ must be increasing which occurs on $\boxed{(0,35) \cup (45,50)}$

b. Find the average acceleration of the car over the interval $0 \le t \le 50$. Indicate units of measure.

$$\text{Avg Acceler} = \frac{v(0) - v(50)}{0 - 50} = \frac{0 - 72}{-50} = \frac{-72}{-50} = \boxed{1.44 \text{ ft/sec}^2}$$

c. Find one approximation for the acceleration of the car at $t = 40$. Show the computations you used to arrive at your answer. Indicate units of measure.

$$v'(40) = a(40) \approx \frac{v(35) - v(45)}{35 - 45} \approx \frac{81 - 60}{-10} \approx \boxed{-2.1 \text{ ft/sec}^2}$$

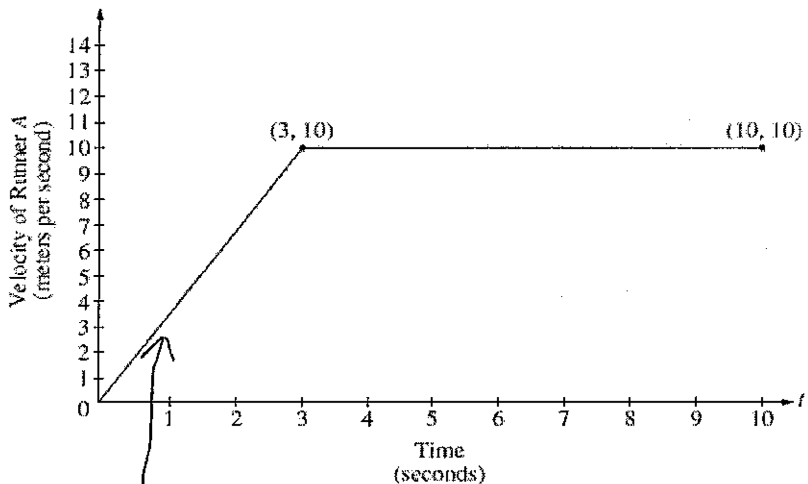
d. Is the speed of the car increasing or decreasing at $t = 40$? Give a reason for your answer.

- $v(40) = 75 > 0$
- Since $v(t)$ is decr. at $t = 40$, then $v'(40) = a(40) < 0$.

* Since $v(40)$ and $a(40)$ have opposite signs, then the speed is decreasing at $t = 40$.

2) 2000 AP Calculus AB #2 (Partial)

Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.



a. Find the velocity of Runner A and the velocity of Runner B at $t = 2$ seconds. Indicate units of measure.

Runner A: $\underbrace{v(t) = \frac{10}{3}t + 0}_{\text{eqn of line}} \rightarrow v(2) = \frac{10}{3}(2) = \boxed{\frac{20}{3} \text{ m/sec}}$

Runner B: $v(t) = \frac{24t}{2t+3} \rightarrow v(2) = \frac{24(2)}{2(2)+3} = \boxed{\frac{48}{7} \text{ m/sec}}$

b. Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

Runner A: $v(t) = \frac{10}{3}t$

$v'(t) = \frac{10}{3}$

$v'(2) = \frac{10}{3}$

$\boxed{a(2) = \frac{10}{3} \text{ m/sec}^2}$

Runner B: $v(t) = \frac{24t}{2t+3}$

$v'(t) = \frac{(2t+3)(24) - (24t)(2)}{(2t+3)^2}$

$v'(2) = \frac{(2(2)+3)(24) - (24(2))(2)}{(2(2)+3)^2}$

$= \frac{168 - 96}{49} = \frac{72}{49}$

$\boxed{a(2) = \frac{72}{49} \text{ m/sec}^2}$

3) 2002 AP Calculus AB #3 (Partial)

An object moves along the x – axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by the function $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

a. What is the acceleration of the object at time $t = 4$?

$$v(t) = \sin\left(\frac{\pi}{3}t\right)$$

$$v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right)$$

$$v'(4) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}\cdot 4\right)$$

$$a(4) = \frac{\pi}{3}\left(-\frac{1}{2}\right) = \boxed{\frac{-\pi}{6}}$$

b. Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is decreasing.

Are either or both of these statements correct? For each statement, provide a reason why it is correct or not correct.

Statement I: since $v'(4) = a(4) < 0$, then the velocity is decreasing at $t = 4$ which is on $3 < t < 4.5$. Hence, Statement I is True.

Statement II: $v(4) = \sin\left(\frac{\pi}{3}\cdot 4\right) = -\frac{\sqrt{3}}{2}$

Since $v(4) < 0$ and $v'(4) = a(4) < 0$, then the speed of the particle is increasing. Hence, Statement II is FALSE.

A particle moves along the x axis such that its position, for $t > 0$, is given by the function $p(t) = e^{2t} - 5t$. Use this information to complete exercises 4 – 7.

4. What are the values of $p'(2)$ and $p''(2)$? Explain what each value represents.

$p'(t) = 2e^{2t} - 5$ $\quad$ $p'(2) = 2e^{2(2)} - 5 = 2e^4 - 5 = \boxed{104.196}$

p'(t) represends the velocity.

$p''(t) = 4e^{2t}$ $\quad$ $p''(2) = 4e^{2(2)} = 4e^4 = \boxed{218.393}$

p''(2) represents the acceleration

5. Based on the values found in part (a), what can be concluded about the speed of the particle at $t = 2$? Give a reason for your answer.

Since $p'(2) = v(t) > 0$ and $p''(2) = a(2) > 0$, then the speed of the particle is increasing at $t = 2$.

6. On what interval(s) of t is the particle moving to the left? To the right? Justify your answers.

$p'(t) = 0$

$2e^{2t} - 5 = 0$

$e^{2t} = 5/2$

$\ln(5/2) = 2t$

$t = 0.458$

p'(t): 0 — neg — .458 — pos. (0.2, 1)

The particle is moving to the left on $(0, 0.458)$ b/c $p'(t) < 0$.

The particle is moving to the Right on $(.458, \infty)$ b/c $p'(t) > 0$.

7. Does the particle ever change directions? Justify your answer.

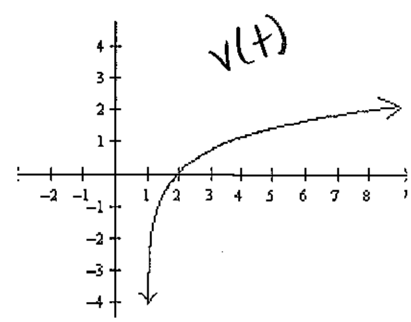
Since $p'(t) = v(t)$ changes signs at $t = 0.458$, the particle changes directions.

8. The graph of $v(t)$, the velocity of a moving particle, is given below. What conclusions can be made about the movement of the particle along the x – axis and the acceleration, $a(t)$, of the particle for $t > 0$? Give reasons for your answers.

* The particle is moving to the left on $(0, 2)$ b/c $v(t) < 0$

* The particle is moving to the Right on $(2, \infty)$ b/c $v(t) < 0$

* Since $v(t)$ is always increasing, then $a(t)$ is always positive for $t > 0$.



9. If the position of a particle is defined by the function $x(t) = t^3 - 9t^2 + 24t$ for $t > 0$, is the speed of the particle increasing or decreasing when $t = 2.5$? Justify your answer.

$x'(t) = v(t) = 3t^2 - 18t + 24$

$v(2.5) = 3(2.5)^2 - 18(2.5) + 24 = -2.25$

$x''(t) = a(t) = 6t - 18$

$a(2.5) = 6(2.5) - 18 = -3$

Since both $v(2.5)$ and $a(2.5) < 0$, the speed of the particle is increasing at $t = 2.5$.

The position of a particle is given by the function $p(t) = (2t-3)e^{2-t}$ for $t > 0$. Answer questions 10 – 12.

10. What is the average velocity from $t = 1$ to $t = 3$?

$$\frac{p(1) - p(3)}{1-3} = \frac{-2.718 - 1.104}{-2} = \boxed{1.911}$$

11. Find an equation for $v(t)$, the velocity of the particle.

$p'(t) = v(t) = (2)(e^{2-t}) + (2t-3)(e^{2-t})(-1)$

$v(t) = 2e^{2-t} - (2t-3)e^{2-t}$

$v(t) = e^{2-t}(2 - 2t + 3)$

$\boxed{v(t) = e^{2-t}(5-2t)}$

12. For what value(s) of t will the $v(t) = 0$?

$v(t) = 0$

$e^{2-t}(5-2t) = 0$

$\downarrow$ $\quad\quad$ $\downarrow$

$e^{2-t} \neq 0$ $\quad\quad$ $5 - 2t = 0$

$\ln 0 = 2-t$ $\quad\quad$ $-2t = -5$

$t = \text{undef.}$ $\quad\quad$ $\boxed{t = 5/2}$

2003 AP Calculus AB #2 (Partial)

A particle moves along the x – axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

$v(t) = (-t-1)\sin(\tfrac{1}{2}t^2)$

13. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain why or why not.

$$v'(t) = a(t) = (-1)(\sin(\tfrac{1}{2}t^2)) + (-t-1)(\cos(\tfrac{1}{2}t^2))(t)$$

$$a(2) = (-1)(\sin(\tfrac{1}{2}(2))^2) + (-2-1)(\cos(\tfrac{1}{2}(2)^2)(2)$$

$$= -.909 + 2.497$$

$$\boxed{a(2) = 1.588}$$

$v(2) = -(2+1)\sin(\frac{2^2}{2})$

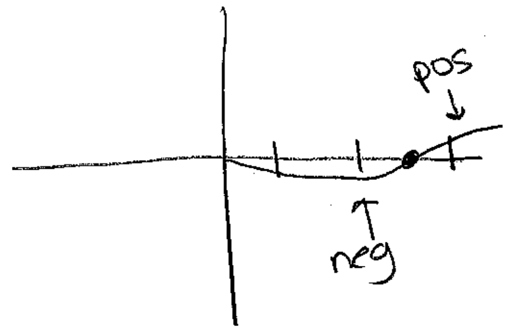
$\boxed{v(2) = -2.728}$

Since $v(2) < 0$ & $a(2) > 0$, then the speed is decr. at $t = 2$

14. Find all times in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

Xmin Xmax

$$\underbrace{v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)}_{y_1} = \underbrace{0}_{y_2}$$



$$\boxed{t = 2.507}$$

On the interval $(0, 3)$, $v(t)$ changes from negative to positive (changing directions) when $t = 2.507$