**AP Calculus**
**Unit 5 – Applications of the Derivative – Part 2**

# Day 8 Notes: Solving Optimization Problems

## General Approach to Solving Optimization Problems:

→ f'(x) changes from ⊕ → ⊖ or ⊖ → ⊕

1. Determine the quantity that is to be maximized or minimized.

2. Draw a picture, define a variable, or use some formula to identify the quantities not valued.

3. Write a primary equation that represents the quantity that is to be optimized. (This equation may or may not contain more than one variable.)

4. If the primary equation contains more than 1 variable, a secondary equation will need to be written that involves the same variables so that one variable can be isolated to show a relationship between the variables.

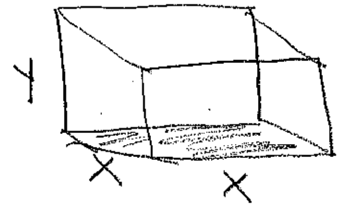
5. Substitute the result of the secondary equation into the primary equation, if necessary, and then differentiate the primary equation to find the maximum/minimum value desired.

**Example 1**

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume? What is the maximum volume?

Dimensions: 6in by 6in by 3in

Max vol = 6x6x3 = 108 in$^3$



$V = x^2 \circled{y}$

$V(x) = x^2\left(\frac{27}{x} - \frac{x}{4}\right)$

$\boxed{V(x) = 27x - \frac{1}{4}x^3}$

$V'(x) = 27 - \frac{3}{4}x^2 = 0$

$-\frac{3}{4}x^2 = -27$

$x^2 = 36$

$x = \pm 6$

$SA = x^2 + 4xy$

$108 = x^2 + 4xy$ { solve for y

$108 - x^2 = 4xy$

$\frac{108 - x^2}{4x} = y$

$\left(\frac{27}{x} - \frac{x}{4} = y\right)$

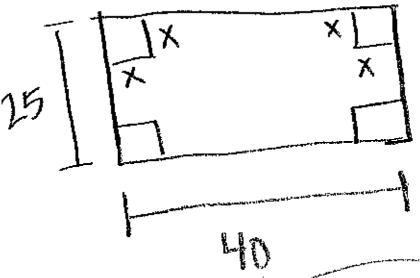
2 7
0 + 6 −

Volume is maximized when x=6 since V'(x) changes from ⊕ → ⊖.

**Example 2**

A box is to be built from a rectangular piece of cardboard that is 25 cm wide and 40 cm long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square which will produce a container that will hold the most amount of soup.

maximize



$V=(x)(25-2x)(40-2x)$

$V=(x)(1000-130x+4x^2)$

$V=1000x-130x^2+4x^3$

$V'(x)=1000-260x+12x^2$

$\frac{12x^2-260x+1000=0}{4}$

$3x^2-65x+250=0$

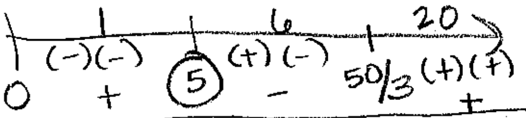
$(3x^2-50x)-15x+250$

$x(3x-50)-5(3x-50)$

$(x-5)(3x-50)$

$x=5 \quad x=50/3$

750, -50, -15

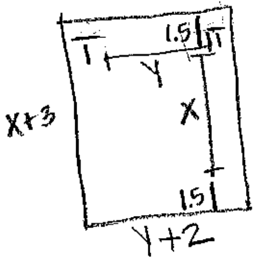


volume is maximized when the square is 5 cm by 5 cm b/c V'(x) changes from ⊕ to ⊖ when x=5.

**Example 3**

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1 ½ inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

minimize



$A=(y+2)(x+3)$

$xy=24$

$y=\frac{24}{x}$

$A=(\frac{24}{x}+2)(x+3)$

$A=24+72x^{-1}+2x+6$

$A'(x)=-72x^{-2}+2$

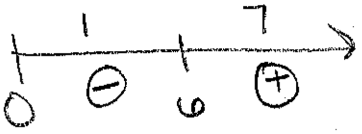
$\frac{-72}{x^2}+2=0$

$\frac{-72}{x^2}=-2$

$-72=-2x^2$

$36=x^2$

$x=\pm 6$



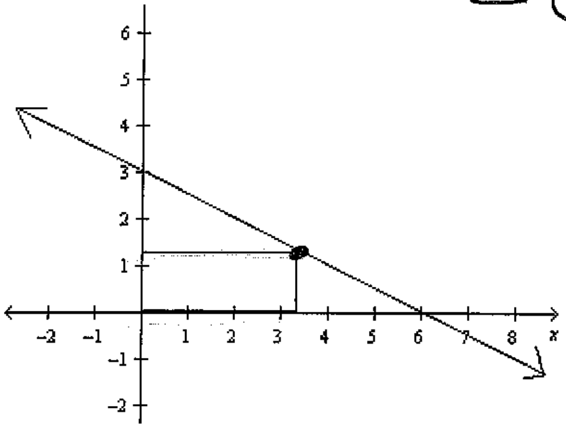
$\frac{24}{6}+2$ 6+3

6 in by 9 in

Area of the paper is a minimum when x=6 b/c A'(x) changes from ⊖ → ⊕

## Example 4

A rectangle is bounded by the *x* and *y* axes and the graph of $y = 3 - ½x$. What length and width should the rectangle have so that its area is a maximum?



$A = xy$

$y = 3 - \frac{1}{2}x$

$A(x) = x(3 - \frac{1}{2}x)$

$A(x) = 3x - \frac{1}{2}x^2$

$A'(x) = \boxed{3-x}$

$3 - x = 0$

$-x = -3$

$x = 3$

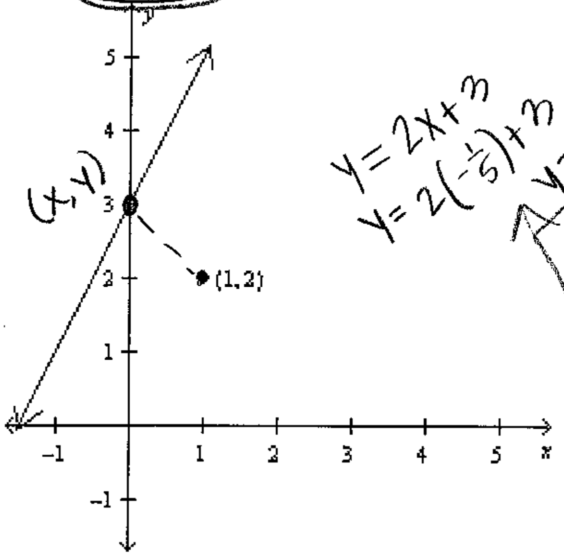
1 4

0 ⊕ 3 ⊖

Since A'(x) changes from ⊕ → ⊖ at x = 3, then the area of the rectangle is a maximum.

## Example 5

Determine the point on the line $y = 2x + 3$ so that the distance between the line and the point (1, 2) is a minimum.



$y = 2x + 3$

$y = 2(-\frac{1}{5}) + 3$

$y = \frac{13}{5}$

$\boxed{(-\frac{1}{5}, \frac{13}{5})}$

$(2x+1)(2x+1)$

$4x^2 + 4x + 1$

$d = \sqrt{(x-1)^2 + (y-2)^2} \leftarrow y = 2x + 3$

$d = \sqrt{(x-1)^2 + (2x+3-2)^2}$

$d = \sqrt{x^2 - 2x + 1 + 4x^2 + 4x + 1}$

$d = \sqrt{5x^2 + 2x + 2}$

$d(x) = (5x^2 + 2x + 2)^{1/2}$

$d'(x) = \frac{1}{2}(5x^2 + 2x + 2)^{-1/2}(10x + 2)$

$= \frac{5x+1}{\sqrt{5x^2+2x+2}} = 0$

$5x + 1 = 0$

$x = -\frac{1}{5}$

-1 0

⊖ -1/5 ⊕

The distance is at a minimum when x = -1/5 b/c d'(x) changes from ⊖ → ⊕.