AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 8 Notes: Solving Optimization Problems

General Approach to Solving Optimization Problems:

1. Determine the quantity that is to be maximized or minimized.

2. Draw a picture, define a variable, or use some formula to identify the quantities not valued.

3. Write a primary equation that represents the quantity that is to be optimized. (This equation may or may not contain more than one variable.)

4. If the primary equation contains more than 1 variable, a secondary equation will need to be written that involves the same variables so that one variable can be isolated to show a relationship between the variables.

5. Substitute the result of the secondary equation into the primary equation, if necessary, and then differentiate the primary equation to find the maximum/minimum value desired.

Example 1

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume? What is the maximum volume?

Example 2

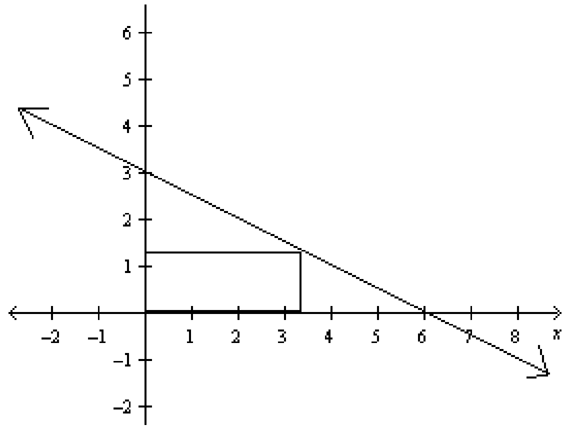
A box is to be built from a rectangular piece of cardboard that is 25 cm wide and 40 cm long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square which will produce a container that will hold the most amount of soup.

Example 3

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1 ½ inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

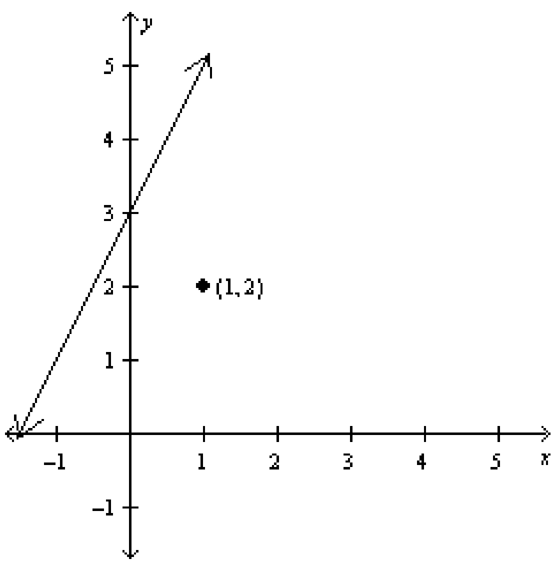
Example 4

A rectangle is bounded by the *x* and *y* axes and the graph of $y = 3 - \frac{1}{2}x$. What length and width should the rectangle have so that its area is a maximum?



Example 5

Determine the point on the line $y = 2x + 3$ so that the distance between the line and the point (1, 2) is a minimum.



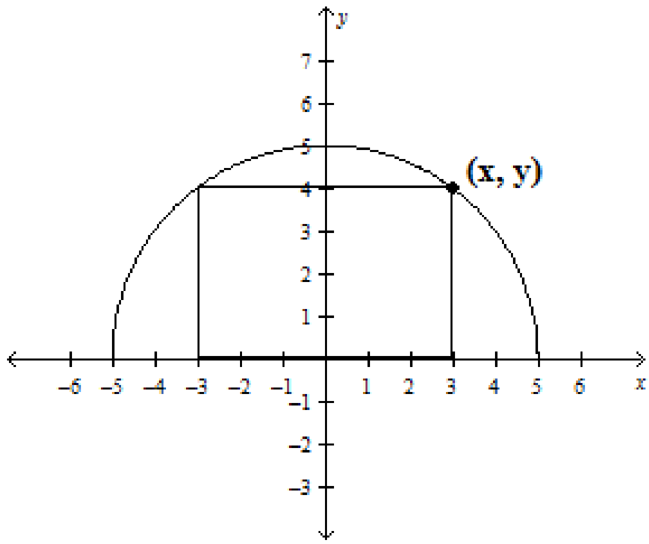
AP Calculus AB **Name: ______________________________**
Unit 5 – Day 8 – Assignment

1. Find the point on the graph of $f(x)=\sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.

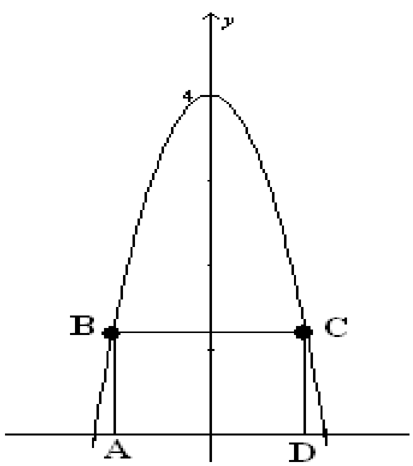
2. A rancher has 200 total feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should each corral be so that the enclosed area will be a maximum?

3. The area of a rectangle is 64 square feet. What dimensions of the rectangle would give the smallest perimeter?

4. A rectangle is bound by the x – axis and the graph of a semicircle defined by $y = \sqrt{25 - x^2}$.
What length and width should the rectangle have so that its area is a maximum?



5. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ as shown in the figure below. Find the x and y coordinates of the point C so that the area of the rectangle is a maximum.



6. Find the maximum volume of a box that can be made by cutting squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides.

7. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must the height, in inches, of the can be?