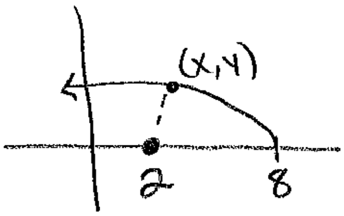
AP Calculus AB **Name:** Answer Key*

Unit 5 – Day 8 – Assignment

1. Find the point on the graph of $f(x)=\sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.

minimize Dist.



$\leftarrow y=\sqrt{-x+8}$

$d=\sqrt{(x-2)^2+(y-0)^2}$

$d=\sqrt{(x-2)^2+(\sqrt{-x+8})^2}$

$d=\sqrt{x^2-4x+4+-x+8}$

$d=\sqrt{x^2-5x+12}$

$d(x)=(x^2-5x+12)^{1/2}$

$d'(x)=\frac{1}{2}(x^2-5x+12)^{-1/2}(2x-5)$

$\frac{2x-5}{2\sqrt{x^2-5x+12}}=0$

$2x-5=0$

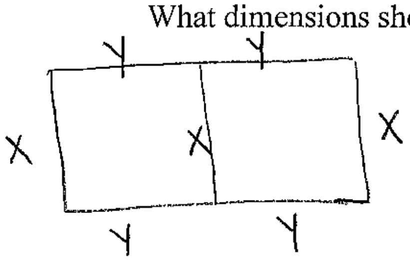
$x=5/2$

0: $\ominus$ | 5/2 | 3: $\oplus$

Since $d'(x)$ changes from $\ominus \to \oplus$ at $x=5/2$, the dist. is a minimum.

Point: $\left(\frac{5}{2},\sqrt{\frac{11}{2}}\right)$

2. A rancher has 200 total feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should each corral be so that the enclosed area will be a maximum?



$200=3x+4y$

$\frac{200-3x}{4}=y$

$A=2xy$

$A(x)=2x\left(\frac{200-3x}{4}\right)$

$A(x)=100x-\frac{3}{2}x^2$

$A'(x)=100-3x$

$100-3x=0$

$x=100/3$

$200=3\left(\frac{100}{3}\right)+4y$

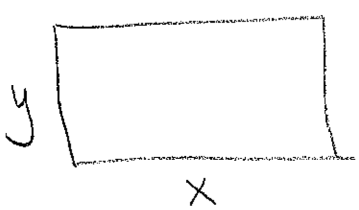
0 | 30: $\oplus$ | $\frac{100}{3}$ | 35: $\ominus$

Since $A'(x)$ changes from $\oplus \to \ominus$ at $x=100/3$, area is at a maximum.

$\frac{100}{3}$ ft by 25 ft

3. The area of a rectangle is 64 square feet. What dimensions of the rectangle would give the smallest perimeter?

minimize perimeter



$64=xy$

$\frac{64}{x}=y$

$P=2x+2y$

$P=2x+2\left(\frac{64}{x}\right)$

$P=2x+128x^{-1}$

$P'(x)=2+-128x^{-2}$

$2-\frac{128}{x^2}=0$

$\frac{128}{x^2}=2$

$2x^2=128$ $x^2=64$ $x=\pm 8$

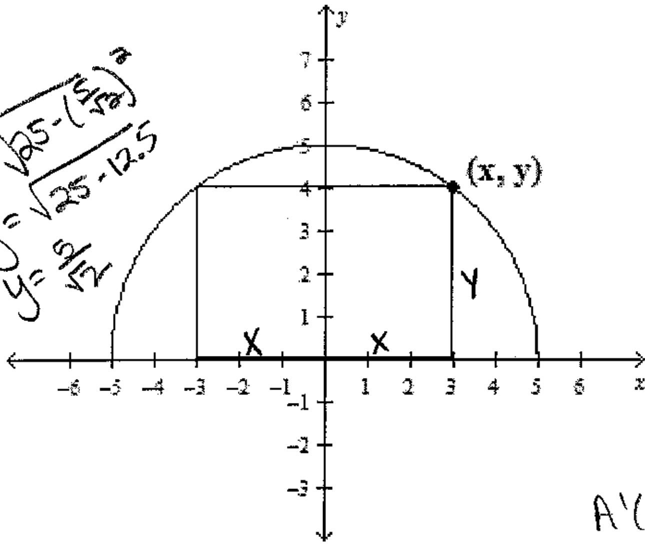
0 | 7: $\ominus$ | 8 | 9: $\oplus$

Since $P'(x)$ changes from $\ominus$ to $\oplus$, $P(x)$ is at a minimum when $x=8$.

8 ft by 8 ft

$\boxed{\text{length} = \frac{10}{\sqrt{2}} \text{ \& width} = \frac{5}{\sqrt{2}}}$

4. A rectangle is bound by the x – axis and the graph of a semicircle defined by $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



$x = \frac{5(2)}{\sqrt{2}}$ $y = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}$

$y = \sqrt{25 - 12.5}$

$y = \frac{5}{\sqrt{2}}$

$A = 2xy \leftarrow y = \sqrt{25 - x^2}$

$A = 2x\left(\sqrt{25 - x^2}\right)$

$A = 2x(25 - x^2)^{1/2}$

$A'(x) = (2)(25 - x^2)^{1/2} + (2x)\frac{1}{2}(25 - x^2)^{-1/2}(-2x)$

$A'(x) = 2(25 - x^2)^{1/2} - \frac{2x^2}{(25 - x^2)^{1/2}}$

$A'(x) = \frac{2(25 - x^2) - 2x^2}{(25 - x^2)^{1/2}}$

$A'(x) = \frac{50 - 4x^2}{\sqrt{25 - 4x^2}} = 0$

$50 - 4x^2 = 0$

$50 = 4x^2$

$x^2 = \frac{25}{2}$

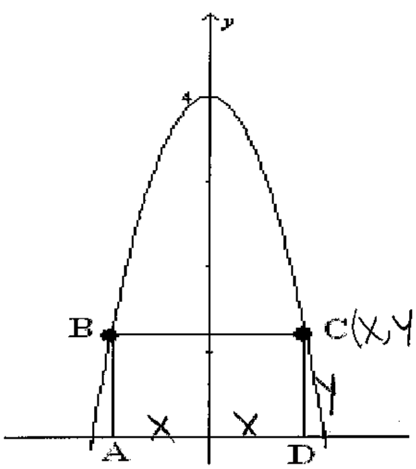
$x = \frac{5}{\sqrt{2}}$

0 (+) $\frac{5}{\sqrt{2}}$ (−)

1 4

A(x) has a maximum at $x = 5/\sqrt{2}$ b/c A'(x) goes from (+) → (−)

5. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ as shown in the figure below. Find the x and y coordinates of the point C so that the area of the rectangle is a maximum.



$A = 2xy \leftarrow y = -4x^2 + 4$

$A = 2x(-4x^2 + 4)$

$A = -8x^3 + 8x$

$A'(x) = -24x^2 + 8$

$-24x^2 + 8 = 0$

$-24x^2 = -8$

$x^2 = \frac{1}{3}$

$x = \sqrt{1/3}$

0 (+) $\sqrt{1/3}$ (−)

1/4 1

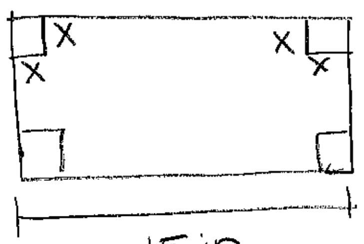
A(x) has a maximum at $x = \frac{1}{3}$ b/c A'(x) changes from (+) → (−).

$x = \sqrt{1/3}$ $y = -4\left(\sqrt{\frac{1}{3}}\right)^2 + 4$

$y = \frac{8}{3}$

$\left(\sqrt{\frac{1}{3}}, \frac{8}{3}\right)$

6. Find the maximum volume of a box that can be made by cutting squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides.



$V = (x)(15-2x)(8-2x)$

$V = (x)(120-46x+4x^2)$

$V = 4x^3 - 46x^2 + 120x$

$V'(x) = 12x^2 - 92x + 120$

$\underline{12x^2 - 92x + 120 = 0}$
$\quad 4$

$3x^2 - 23x + 30 = 0$

$(3x^2 - 18x) - 5x + 30$

$3x(x-6) - 5(x-6)$

$(3x-5)(x-6)$

$x = 5/3 \quad x = 6$

$\frac{90}{-18 \mid -5}$

1 | 5 | 7
(−)(−) | (+)(−) | (+)(+)
0 ⊕ $\frac{5}{3}$ ⊖ 6 ⊕

$V(t)$ is a maximum at $x = 5/3$ b/c $V'(t)$ changes from ⊕ → ⊖.

$\frac{5}{3}$ in by $\frac{35}{3}$ in by $\frac{14}{3}$ in $\boxed{V = 90.741 \text{ in}^3}$

7. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must the height, in inches, of the can be?

$A = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

$16\pi = \pi r^2 h$

$\frac{16\pi}{\pi r^2} = h$

$h = \frac{16}{r^2}$

$A = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$A = 2\pi r^2 + 32\pi r^{-1}$

$A'(x) = 4\pi r - 32\pi r^{-2}$

$4\pi r - \frac{32\pi}{r^2} = 0$

$-\frac{32\pi}{r^2} = -4\pi r$

$-32\pi = -4\pi r^3$

$8 = r^3$

$r = 2$

1 | 3
0 ⊖ 2 ⊕

Surface Area is a minimum when $r = 2$ b/c $A'(x)$ goes from ⊖ → ⊕

$h = \frac{16}{r^2} = \frac{16}{2^2} = 4$

$\boxed{\text{Height} = 4 \text{ in}}$