AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 7 Notes: Free Response Practice with Properties of Motion

NO CALCULATOR

A particle moves along the x – axis with velocity at time $t \geq 0$ given by $v(t) = -1 - e^{1-t}$.

a. Find the acceleration of the particle at $t = 3$.

b. Is the speed of the particle increasing at $t = 3$? Give a reason for your answer.

c. Find all values of t at which the particle changes direction. Justify your answer.

d. The function $p(t) = e^{1-t} - t$ models the position of the particle for $t \geq 0$. Find the total distance that particle traveled on the time interval $0 \leq t \leq 3$.

---

a) $v'(t) = a(t)$

$v'(t) = -e^{1-t}(-1) = e^{1-t}$

$a(3) = e^{1-3} = e^{-2} = \boxed{\frac{1}{e^2}}$

b) $v(3) = -1 - e^{1-3} = -1 - e^{-2} = -1 - \frac{1}{e^2}$

$v(3) < 0$

$a(3) > 0$ (from part a)

Since $v(3) < 0$ & $a(3) > 0$, then the speed is decreasing at $t = 3$.

c) $v(t) = 0$ [when particle changes direction]

$-1 - e^{1-t} = 0$

$-e^{1-t} = 1$

$e^{1-t} = -1$

$\ln(-1) = 1 - t$

No solution

Since $v(t) \neq 0$, the particle never changes directions.

d) Total Distance = $|p(0) - p(3)|$

same as net distance since no change in direction.

$p(0) = e^{1-0} - 0$
$= e^1 - 0$
$= e$

$p(3) = e^{1-3} - 3$
$= e^{-2} - 3$

$|p(0) - p(3)| =$

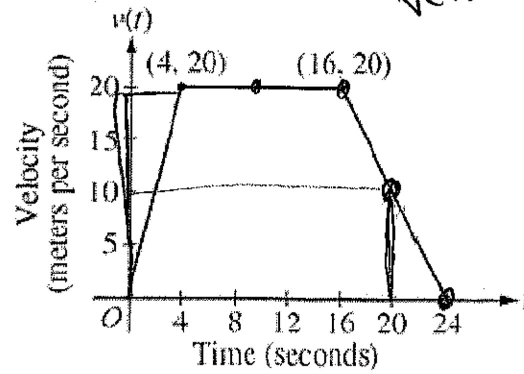
$|e - (e^{-2} - 3)| =$

$\boxed{|e - e^{-2} + 3|}$

NO CALCULATOR

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity, $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.

v(t)



a. For what interval(s) of time does the car have zero acceleration? Show the work and explain the analysis that leads to your answer.

b. For each value of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

c. Let $a(t)$ be the car's acceleration at time t in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

d. Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

---

a) $a(t) = v'(t) = 0$ when the graph of $v(t)$ is constant.

$\boxed{(4, 16)}$

b) v'(4) does not exist b/c the graph of v(t) at t = 4 has a cusp making v(t) not differentiable at t = 4.

$v'(20) = \dfrac{20 - 0}{16 - 24} = \dfrac{20}{-8} = \boxed{\dfrac{-5}{2}\ \text{m/sec}^2}$

c)

$$a(t) = \begin{cases} 5, & 0 < t < 4 \\ 0, & 4 < t < 16 \\ -5/2, & 16 < t < 24 \end{cases}$$

*slope of line segment

d) Avg. Rate of Change $= \dfrac{v(8) - v(20)}{8 - 20}$

$= \dfrac{20 - 10}{8 - 20} = \dfrac{10}{-12}$

$= \boxed{\dfrac{-5}{6}\ \text{m/sec}^2}$

Since v(t) is <u>not</u> differentiable on the interval $8 < t < 20$ (cusp at $t = 16$), the M.V.T does not guarantee a value of c such that $v'(c) = -\frac{5}{6}$.

$p(t) = t^3 - 4t^2 - 3t + 1$
$v(t) = 3t^2 - 8t - 3$
$a(t) = 6t - 8$

NO CALCULATOR PERMITTED

A particle moves along the x – axis so that its position at any time $t \geq 0$ is given by the function $p(t) = t^3 - 4t^2 - 3t + 1$, where p is measured in feet and t is measured in seconds.

a. Find the average velocity on the interval $t = 1$ and $t = 2$ seconds. Give your answer using correct units.

b. On what interval(s) of time is the particle moving to the left? Justify your answer.

c. Using appropriate units, find the value of $p'(3)$ and $p''(3)$. Based on these values, describe the motion of the particle at $t = 3$ seconds. Give a reason for your answer.

d. What is the maximum velocity on the interval from $t = 1$ to $t = 3$ seconds. Show the analysis that leads to your conclusion.

e. Find the total distance that the particle moves on the interval [1, 5]. Show and explain your analysis.

(a) Avg. Velocity $= \dfrac{p(1) - p(2)}{1-2} = \dfrac{-5-(-13)}{-1} = \boxed{-8 \text{ ft/sec}}$

$p(1) = (1)^3 - 4(1)^2 - 3(1) + 1 = -5$
$p(2) = (2)^3 - 4(2)^2 - 3(2) + 1 = -13$

(b) particle moves to left when $v(t) = p'(t) < 0$

$p'(t) = 3t^2 - 8t - 3$

$3t^2 - 8t - 3 = 0$ $\frac{-9}{-9 \mid 1}$

$(3t^2 - 9t) + (1t - 3)$

$3t(t-3) + 1(t-3)$

$(3t+1)(t-3) = 0$

can't have neg. time → $t = \cancel{-1/3}$ $t = 3$

1   4

0 (+)(-) neg (3) (+)(+) pos.

$\boxed{\text{particle moves to the left on } (0,3) \text{ b/c } v(t) < 0.}$

(c) $p'(t) = 3t^2 - 8t - 3$

$p'(3) = 3(3)^2 - 8(3) - 3 =$

$\boxed{p'(3) = 0 \text{ ft/sec}}$

$p''(t) = 6t - 8$

$p''(3) = 6(3) - 8$

$\boxed{p''(3) = 10 \text{ ft/sec}^2}$

Since $p'(3) = v(3) = 0$ and $p''(3) = a(3) \neq 0$, then at $t = 3$ the particle is changing directions.

(d) $v'(t) = 0 \rightarrow 6t - 8 = 0 \quad t = \frac{4}{3}$

$v(1) = 3(1)^2 - 8(1) - 3 = -8$ ft/sec

$v(3) = 3(3)^2 - 8(3) - 3 = 0$ ft/sec

$v(4/3) = 3(\frac{4}{3})^2 - 8(\frac{4}{3}) - 3$

$= 3(\frac{16}{9}) - 8(\frac{4}{3}) - 3$

$= \frac{48}{9} - \frac{32}{3} - 3$

$= \frac{48}{9} - \frac{96}{9} - \frac{27}{9} = \frac{-75}{9} = -\frac{25}{3}$ ft/sec

(d) (continued)

$\boxed{\text{The Extreme Value Theorem guarantees the maximum velocity on } [1,3] \text{ is } 0 \text{ ft/sec.}}$

(e) Total Distance = $|p(1) - p(3)| + |p(3) - p(5)|$

From part (c), at $t=3$ the particle is changing directions.

$p(1) = (1)^3 - 4(1)^2 - 3(1) + 1 = -5$

$p(3) = (3)^3 - 4(3)^2 - 3(3) + 1 = -17$

$p(5) = (5)^3 - 4(5)^2 - 3(5) + 1 = 11$

$|-5 - (-17)| + |-17 - 11|$

$= 12 + 28$

$= \boxed{40 \text{ feet}}$

CALCULATOR PERMITTED

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per min)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Find the average acceleration on the interval $5 \le t \le 20$. Express your answer using correct units of measure.

b. Based on the values in the table, on what interval(s) is the acceleration of the plane guaranteed to equal zero on the open interval $0 < t < 40$? Justify your answer.

$a(t) = 0$
$v'(t) = 0$
Rolle's Thm

c. Does the data represent velocity values of the plane moving away from its point of origin or returning to its point of origin? Give a reason for your answer.

d. The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? What does this value indicate about the velocity at $t = 23$? Justify your answer, indicating units of measure.

(a) Avg. Acceleration $= \dfrac{V(5) - V(20)}{5-20} = \dfrac{9.2-4.5}{-15} = \boxed{-0.313 \text{ miles}/\text{min}^2}$

(b) → since $V(t)$ is continuous & differentiable on $0<t<40$ &

$V(0) = 7.0$ $\quad$ $V(25) = 2.4$
$V(15) = 7.0$ $\quad$ $V(30) = 2.4$

∴ Rolle's Thm guarantees that on the interval $(0,15)$ & $(25,30)$ there is a value such that $a(t) = 0$

(c) Since all values of $V(t) > 0$, the distance the plane is from its point of origin is increasing so the plane is moving away from point of origin.

(d) $f'(23) = \boxed{-0.408 \text{ miles}/\text{min}^2}$

use MATH 8

Since $f'(23) < 0$, the velocity is decreasing at $t = 23$ minutes.