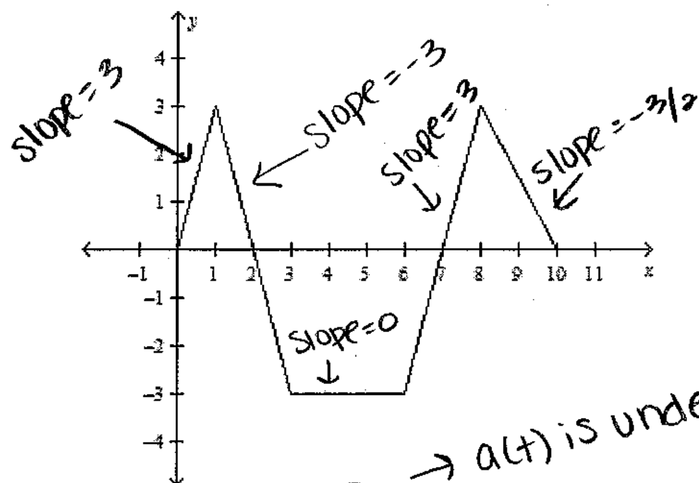
AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 6 Notes: More on Particle Motion (Finding Net & Total Distance)

The graph below represents the velocity, $v(t)$ which is measured in meters per second, of a particle moving along the x – axis.



→ a(t) is undefined

At what value(s) of t does the particle have no acceleration on the interval (0, 10)? Justify your answer.

a(t) is undefined when v(t) is not differentiable. Since v(t) has a cusp at t = 1, t = 3, t = 6, & t = 8, there is no acceleration at these times.

Express the acceleration, $a(t)$, as a piecewise-defined function on the interval (0, 10).

eqn of $v(t) = 3x + 0$
$a(t) = 3$

$$a(t) = \begin{cases} 3, & 0 < t < 1 \\ -3, & 1 < t < 3 \\ 0, & 3 < t < 6 \\ 3, & 6 < t < 8 \\ -3/2, & 8 < t < 10 \end{cases}$$

For what value(s) of t is the particle moving to the right? To the left? Justify your answer.

Right when $v(t) > 0 \rightarrow (0, 2) \cup (7, 10)$

Left when $v(t) < 0 \rightarrow (2, 7)$

Find the average acceleration of the particle on the interval [1, 8]. Show your work.

$$\text{avg acceleration} = \frac{v(1) - v(8)}{1 - 8} = \frac{3 - 3}{-7} = \boxed{0 \text{ m/s}^2}$$

Definition of Net Distance: The distance from where the object begins and where it ends.

Definition of Total Distance: The sum of all distances moved in any direction

If a particle is moving in the same direction the entire amount of time, what can be said about the net distance and the total distance?

The Net Distance = The TOTAL Distance

To Find the Net Distance a Particle Travels on an Interval

On the interval $[a,b]$, the net distance an object travels is $|p(a)-p(b)|$, where $p(t)$ is the position function.

To Find the Total Distance a Particle Travels on an Interval

→ Provided the object moves the same direction on the interval $[a,b]$, total distance $= |p(a)-p(b)|$

→ But, if the object changes directions at $t=c$ on (a,b) then total distance $=$

$$|p(a)-p(c)| + |p(c)-p(b)|.$$

The position of a particle is given by the function $p(t) = 2t^3 - 6t^2 + 8t$ where $p(t)$ is measured in centimeters. Find the net and total distance the particle travels from $t = 1.5$ seconds to $t = 4$ seconds.

Net Distance $= |p(1.5) - p(4)| = |5.25 - 64| = \boxed{58.75 \text{ cm}}$

$v(t) = 6t^2 - 12t + 8 > 0$ for all values on $(1.5, 4)$

[Look at graph → all above x-axis]

So the particle moves to the right on the entire interval.

$\therefore$ Net Distance = Total Distance = 58.75 cm

The position of a particle is given by the function $p(t) = e^{2t} - 8t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = 0.5$ minutes to $t = 1.5$ minutes.

Net Distance $= |p(0.5) - p(1.5)| = |-1.282 - 8.086| = \boxed{9.368 \text{ ft}}$

$v(t) = 2e^{2t} - 8$ $\quad 2e^{2t} - 8 = 0$

$t = 0.693$ (changes signs)

neg → pos [left → right]

Total Distance $= |p(0.5) - p(0.693)| + |p(0.693) - p(1.5)| =$

$|-1.282 - (-1.545)| + |(-1.545) - 8.086| = \boxed{9.894 \text{ ft}}$

The position of a particle is given by the function $p(t) = t + 2\sin t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = \frac{\pi}{6}$ minutes to $t = \frac{5\pi}{4}$ minutes.

Net Distance $= \left|p\left(\frac{\pi}{6}\right) - p\left(\frac{5\pi}{4}\right)\right| = |1.524 - 2.513| = \boxed{0.989 \text{ feet}}$

$v(t) = 1 + 2\cos t$ $\quad 1 + 2\cos t = 0$

$t = 2.094$ (changes signs)

Total Distance $= \left|p\left(\frac{\pi}{6}\right) - p(2.094)\right| + \left|p(2.094) - p\left(\frac{5\pi}{4}\right)\right| =$

$|1.524 - 3.826| + |3.826 - 2.513| =$

$2.302 + 1.313 = \boxed{3.615 \text{ feet}}$