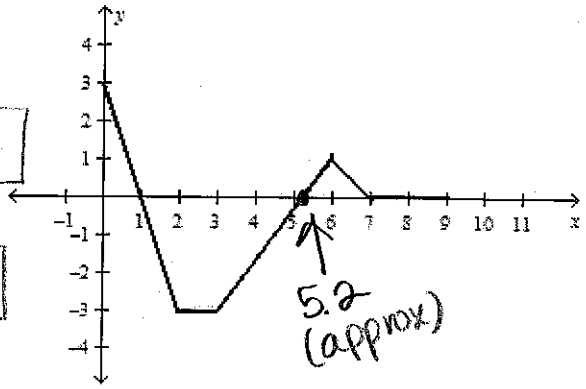
AP Calculus AB
Unit 5 – Day 6 – Assignment

Name: Answer Key *

The function whose graph is pictured below, represents the velocity, $v(t)$, of a particle for $t = 0$ to $t = 9$ seconds moving along the x – axis. Use the graph to complete exercises 1 – 4.

1. On what interval(s) is the particle moving to the right? Left? Justify your answer.



Right → when $v(t) > 0$ → $(0,1) \cup (5.2,7)$

left → when $v(t) < 0$ → $(1, 5.2)$

2. On what interval(s) is the particle slowing down? Speeding up? Justify your answer.

* A particle is slowing down [speed decreases] when $v(t)$ & $a(t)$ have different signs.

→ This occurs when $v(t)$ is decr & above x-axis or $v(t)$ is incr. & below x-axis.

$(0,1) \cup (3,5.2) \cup (6,7)$

* Speeding up = speed increases = $v(t)$ & $a(t)$ same sign

$v(t)$ incr & above x-axis or $v(t)$ decr & below x-axis

$(1,2) \cup (5.2,6)$

3. At what value(s) of t is the particle momentarily stopped and changing directions? Justify your answer.

when $v(t) = 0$ & $a(t) \neq 0$

$v(t)$ crosses the x-axis

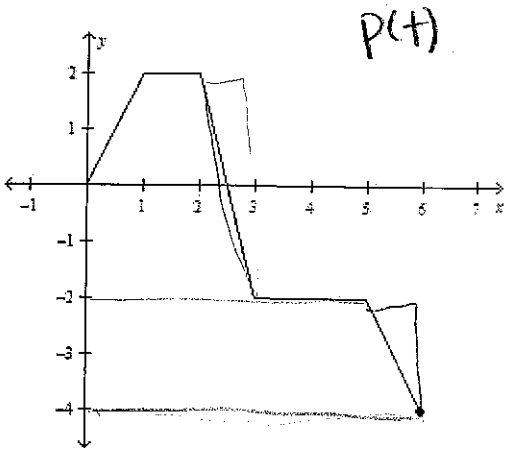
$t = 1$ & $t = 5.2$

4. On what interval of the time is the acceleration 0? Justify your answer.

$A(t) = 0$ anytime the graph of $v(t)$ is constant

$(2,3) \cup (7,9)$

The graph below represents the position, $p(t)$, of a particle that is moving along the x – axis on the interval $0 \le t \le 6$. Use the graph to complete exercises 5 – 9. $p(t)$ is measured in centimeters and t is measured in seconds.



5. For what interval(s) of time is the particle moving to the right? Justify your answer.

p(t) is increasing → particle moving to the Right

(0, 1)

6. For what interval(s) of time is the particle moving to the left? Justify your answer.

p(t) is decreasing → particle moving to the left

(2, 3) ∪ (5, 6)

7. Express the velocity, $v(t)$, as a piecewise-defined function on the interval $0 < t < 6$.

$$v(t) = \begin{cases} 2, & 0 < t < 1 \\ 0, & 1 < t < 2 \\ -4, & 2 < t < 3 \\ 0, & 3 < t < 5 \\ -2, & 5 < t < 6 \end{cases}$$

Slope of each line segment

8. At what value(s) of t is the velocity undefined on the interval $1 < t < 6$? Graphically justify your reasoning.

v(t) is undefined when p(t) is not differentiable

t = 1, t = 2, t = 3, t = 5 [cusp on graph]

9. Find the average velocity of the particle on the interval $1 \le t \le 6$.

$$\text{avg velocity} = \frac{p(1) - p(6)}{1 - 6} = \frac{2 - (-4)}{-5} = \frac{6}{-5} \text{ cm/sec}$$

A particle moves along the $x-$axis so that at any time $0 \le t \le 5$, the velocity, in meters per second, is given by the function $v(t) = (t-2)^2 \cos 2t$. Use a graphing calculator to complete exercises 10 – 13

10. On the interval $0 \le t \le 5$, at how many times does the particle change directions? Give a reason for your answer.

The graph of v(t) crosses the x-axis [or changes signs] at 3 values of t on [0,5].

3 times

11. Using appropriate units, what is the value of $v'(2)$. Describe the motion of the particle at this time. Justify your answer.

use math 8

$v(2) = 0$

$v'(2) = 1.514 \times 10^{-6} =$ **0.000 m/sec²**

Since $v(2) = 0$ and $v'(2) = 0 = a(2)$, the particle is at a dead stand still, no movement at all.

12. Using appropriate units, what is the average acceleration between $t = 1$ and $t = 3.5$ seconds?

$$\text{Avg Acceleration} = \frac{v(1) - v(3.5)}{1 - 3.5} = \frac{-.416 - 1.696}{-2.5} =$$ **0.845 m/sec²**

13. What is the acceleration of the particle the first time that the velocity is 0?

$v(t) = (t-2)^2 \cos 2t = 0$

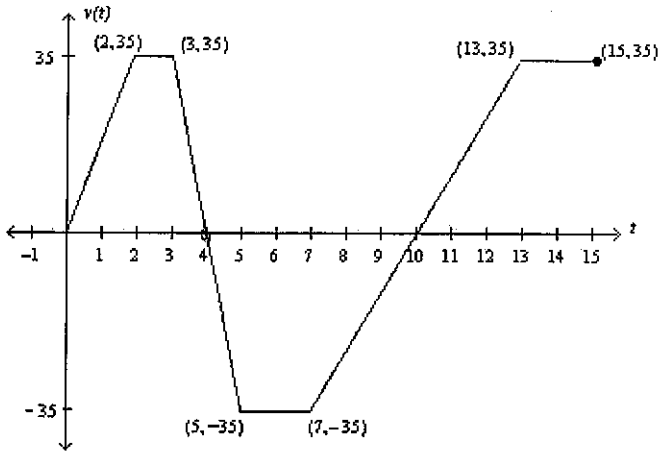
$t = 0.786$ $t = 2.356$ $t = 3.927$

↑ 1st time velocity = 0

use math 8 ↓

$v'(0.786) =$ **−2.945 m/sec²**

Jeff leaves his house riding his bicycle toward school. His velocity $v(t)$, measured in feet per minute, on the interval $0 \le t \le 15$, for t minutes, is shown in the graph to the right. Use the graph to complete exercises 14 – 17.



v(t)

14. Find the value of $v'(4)$. Explain, using appropriate units, what this value represents.

$v'(4) = \frac{35-(-35)}{3-5} = \frac{70}{-2} =$ **-35 ft/min²**

v'(4) represents Jeff's acceleration 4 minutes into his ride to school.

15. On the interval $0 \le t \le 5$, is there any interval of time at which $a(t) = 0$? Explain how you know.

v'(t) = 0

a(t) = v'(t) = 0 when v(t) is constant.

(2,3)

16. On the interval $0 \le t \le 5$, does Rolle's Theorem guarantee that there will be a value of t such that $a(t) = 0$? Justify your answer.

v'(t) = 0
a(t) = 0

Rolle's Thm. does not guarantee a value of t such that a(t) = 0 b/c
① v(t) is not differentiable at t = 2 & t = 3 [cusp]
② v(0) ≠ v(5)

17. At some point, Jeff realizes that he forgot something at home and has to turn around. After how many minutes does he turn around? Give a reason for your answer.

On (0,4), v(t) > 0 which means Jeff's distance from his house is increasing.

At **t = 4**, v(t) changes from positive to negative which means his distance between his location & home begins to decrease.

∴ He turns around at 4 sec.