AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 3 Notes: Applying Theorems in Calculus (Intermediate Value, Extreme Value, Mean Value, Rolle's)

Example 1: The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function *R* of time *t*. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	0	3	6	9	12	15	18	21	24
R(*t*) (gallons per hour)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a. Estimate the value of $R'(5)$, indicating correct units of measure. Explain what this value means about $R(t)$.

b. Using correct units of measure, find the average rate of change of $R(t)$ from $t = 3$ to $t = 18$.

c. Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Example 2: The total order and transportation cost $C(x)$, measured in dollars, of bottles of Pepsi Cola is approximated by the function

$$C(x) = 10{,}000\left(\frac{1}{x} + \frac{x}{x+3}\right),$$

where x is the order size in number of bottles of Pepsi Cola in hundreds. Answer the following questions.

a. Is there guaranteed a value of r on the interval $0 \le r \le 3$ such that the average rate of change of cost is equal to $C'(r)$? Give a reason for your answer.

b. Is there a value of r on the interval $3 \le r \le 6$ such that $C'(r) = 0$. Give a reason for your answer and if such a value of r exists, then find that value of r.

c. For $3 \le x \le 9$, what is the greatest cost for order and transportation?

Example 3: A car company introduces a new car for which the number of cars sold, *S*, is modeled by the function

$$S(t) = 300\left(5 - \frac{9}{t+2}\right),$$

where *t* is the time in months.

a. Find the value of $S'(2.5)$. Using correct units, explain what this value represents in the context of this problem.

b. Find the average rate of change of cars sold over the first 12 months. Indicate correct units of measure and explain what this value represents in the context of this problem.

c. Is it possible that a value of *c* for $0 \le c \le 12$ exists such that $S'(c)$ is equal to the average rate of change? Give a reason for your answer and if such a value of *c* exists, find the value.

Example 4:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by the equation $h(x) = f(g(x)) - 6$.

a. Find the equation of the tangent line drawn to the graph of h when $x = 3$.

b. Find the rate of change of h for the interval $1 < x < 3$.

c. Explain why there must be a value of r for $1 < r < 3$ such that $h(r) = -2$.

d. Explain why there is a value of c for $1 < c < 3$ such that $h'(c) = -5$.

AP Calculus AB **Name: ______________________________**
Unit 5 – Day 3 – Assignment

For the functions in exercises 1 and 2, determine if the Mean Value Theorem holds true for $0 < c < 5$? Give a reason for your answer. If it does hold true, find the guaranteed value(s) of c. **[CALC]**

1. $f(x) = -2 + \frac{1}{2}\lvert x-3\rvert$	
2. $g(x) = -2x + \sin^2 x$	

3. Administrators at a hospital believe that the number of beds in use is given by the function

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50,$$

where t is measured in days. **[CALC]**

a. Find the value of $B'(7)$. Using correct units of measure, explain what this value means in the context of the problem.

b. For $12 \le t \le 20$, what is the maximum number of beds in use?

4. For $t \geq 0$, the temperature of a cup of coffee in degrees Fahrenheit t minutes after it is poured is modeled by the function $F(t) = 68 + 93(0.91)^t$. Find the value of $F'(4)$. Using correct units of measure, explain what this value means in the context of the problem. **[CALC]**

For questions 5 – 8, use the table given below which represents values of a differentiable function g on the interval $0 \leq x \leq 6$. Be sure to completely justify your reasoning when asked, citing appropriate theorems, when necessary.

x	0	2	3	4	6
$g(x)$	–3	1	5	2	1

5. Estimate the value of $g'(2.5)$.

6. If one exists, on what interval is there guaranteed to be a value of c such that $g(c) = -1$? Justify your reasoning.

7. If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 0$? Justify your reasoning.

8. If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 4$? Justify your reasoning.