AP Calculus AB **Name:** Answer Key*
Unit 5 – Day 3 – Assignment

For the functions in exercises 1 and 2, determine if the Mean Value Theorem holds true for $0 < c < 5$? Give a reason for your answer. If it does hold true, find the guaranteed value(s) of c. **[CALC]**

1. $f(x) = -2 + \frac{1}{2}\lvert x-3\rvert$	- f(x) is continuous on [0,5] ✓ - f(x) is not differentiable at x = 3 since the graph has a cusp. → f(x) not diff. on (0,5) ∴ mean value theorem not applicable
2. $g(x) = -2x + \sin^2 x$ $-2x + (\sin x)^2$	- g(x) is continuous on [0,5] ✓ - g(x) is differentiable on (0,5) ✓ $g'(x) = -2 + 2(\sin x)(\cos x)$ $-2 + 2\sin c\cos c = \frac{g(0)-g(5)}{0-5}$ $-2 + 2\sin c\cos c = \frac{0-(-9.080)}{-5}$

$\underbrace{-2+2\sin c\cos c}_{y_1} = \underbrace{-1.816}_{y_2}$

$\boxed{c = 0.093 \quad c = 1.478 \quad c = 3.234 \quad c = 4.620}$

3. Administrators at a hospital believe that the number of beds in use is given by the function

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50,$$

where t is measured in days. **[CALC]**

CALC MATH 8

a. Find the value of $B'(7)$. Using correct units of measure, explain what this value means in the context of the problem.

→ $B'(7) = \boxed{1.530 \text{ beds in use per day}}$

Since $B'(7) > 0$, then on day 7 the number of beds being used is increasing at a rate of 1.530 beds per day.

b. For $12 \le t \le 20$, what is the maximum number of beds in use? → B(t) is continuous on [12, 20] ✓

E.V.T

$B'(t) = 20\cos\left(\frac{1}{10}t\right)\cdot\left(\frac{1}{10}\right)$

$B'(t) = 2\cos\left(\frac{1}{10}t\right)$

$\underbrace{2\cos\left(\frac{1}{10}t\right)}_{y_1} = \underbrace{0}_{y_2}$ $t = 15.708$

$B(12) = 68.641$
$B(20) = 68.186$
$B(15.708) = 70$

$\boxed{\text{The maximum \# of beds in use at any time is 70 beds}}$

4. For $t \geq 0$, the temperature of a cup of coffee in degrees Fahrenheit t minutes after it is poured is modeled by the function $F(t) = 68 + 93(0.91)^t$. Find the value of $F'(4)$. Using correct units of measure, explain what this value means in the context of the problem. **[CALC]**

calc math 8

$F'(4) = -6.015$ °F per minute

Since $F'(4) < 0$, then the temperature of the coffee is decreasing at a rate of 6.015 °F per minute after the coffee is poured.

For questions 5 – 8, use the table given below which represents values of a differentiable function g on the interval $0 \leq x \leq 6$. Be sure to completely justify your reasoning when asked, citing appropriate theorems, when necessary.

x	0	2	3	4	6
$g(x)$	–3	1	5	2	1

5. Estimate the value of $g'(2.5)$.

$g'(2.5) \approx \frac{g(2)-g(3)}{2-3} \approx \frac{1-5}{2-3} \approx \frac{-4}{-1} \approx \boxed{4}$

6. If one exists, on what interval is there guaranteed to be a value of c such that $g(c) = -1$? Justify your reasoning.

I.V.T.

→ Since $g(x)$ is differentiable on $[0,6]$, then it is also continuous on $[0,6]$

Since $g(c) = -1$ is between $g(0) = -3$ and $g(2) = 1$, then according to the Intermediate Value Theorem, a value of c is guaranteed on the interval $\boxed{(0,2)}$.

7. If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 0$? Justify your reasoning.

Rolle's

→ Since $g(x)$ is continuous on $[0,6]$ and differentiable on $(0,6)$ and $g(2)=1$ and $g(6)=1$, then Rolle's Theorem is applicable and guarantees a value of c on $\boxed{(2,6)}$ such that $g'(c)=0$.

8. If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 4$? Justify your reasoning.

M.V.T

→ $g(x)$ is cont. on $[0,6]$ & diff. on $(0,6)$ ✓

Since $\frac{g(2)-g(3)}{2-3} = \frac{1-5}{-1} = 4$, then the mean value Theorem guarantees a value of c on the interval $\boxed{(2,3)}$ such that $g'(c) = 4$.