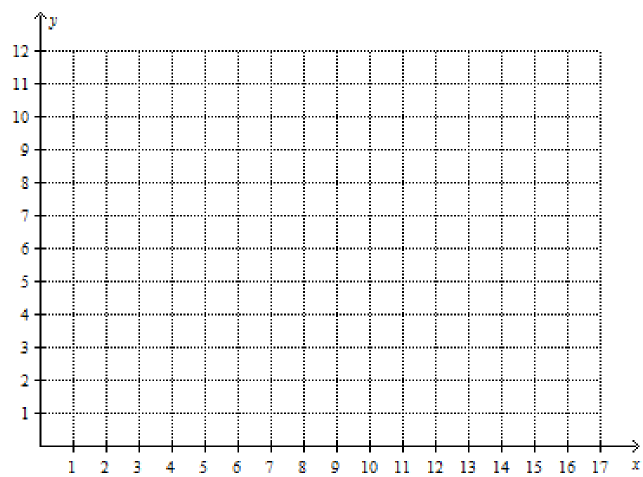
AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 2 Notes: Mean Value Theorem & Rolle's Theorem

Consider the values of a differentiable function, $f(x)$, in the table below to answer the questions that follow. Plot the points and connect them on the grid below.

x	0	2	4	6	8	10	12	14	16
$f(x)$	1	5	8	10	11	10	8	5	1



In calculus, the derivative has many interpretations. One of the most important interpretations is that the derivative represents the Rate of Change of a Function. When speaking of rate of change, there are two rates of change that can be found that are associated with a function—average rate of change and instantaneous rate of change.

Average Rate of Change of $f(x)$ on an Interval

Find the average rate of change of $f(x)$ on the interval [2, 12].

Instantaneous Rate of Change of $f(x)$ at a Point

Is the instantaneous rate of change of at x = 4 greater than the rate of change at $x = 6$? Justify.

Rolle's Theorem

Consider the function, $f(x)$, presented on the previous page. Does Rolle's Theorem apply on the following intervals? Explain why or why not?

Interval [2, 14]	
Interval [2, 10]	

For each of the functions below, determine whether Rolle's Theorem is applicable or not. Then, apply the theorem to find the values of c guaranteed to exist.

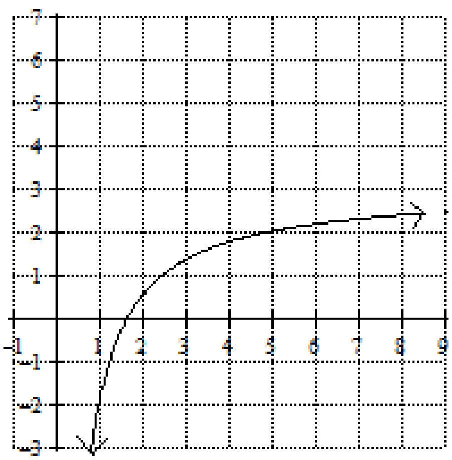
1. $g(x) = 9x^2 - x^4$ on the interval [–3, 0]	2. $g(x) = \frac{\sin 2x}{x+2}$ on the interval [–4, –1]

Rolle's Theorem guarantees that if a function is continuous on the closed interval [*a*, *b*], differentiable on the open interval (*a*, *b*), and *f*(*a*) = *f*(*b*), then there is guaranteed to exist a value of *c* on (*a*, *b*) where the **instantaneous rate of change is equal to zero.**

The Mean Value Theorem is similar. In fact, Rolle's Theorem is a specific case of what is known in calculus as the Mean Value Theorem.

Mean Value Theorem

Consider the function $h(x) = 3 - \frac{5}{x}$. The graph of *h*(*x*) is pictured below. Does the M.V.T. apply on the interval [–1, 5]? Explain why or why not.



Does the M.V.T. apply on the interval [1, 5]? Why or why not?

Graphically, what does the M.V.T. guarantee for the function on the interval [1, 5]? Draw this on the graph to the left.

Apply the M.V.T. to find the value(s) of *c* guaranteed for *h*(*x*) on the interval [1, 5]

Explain why you cannot apply the Mean Value Theorem for $f(x) = x^{\frac{2}{3}} - 2$ on the interval $[-1, 1]$.

Find the equation of the tangent line to the graph of $f(x) = 2x + \sin x + 1$ on the interval $(0, \pi)$ at the point which is guaranteed by the mean value theorem.

The Mean Value Theorem guarantees that if a function is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is guaranteed to exist a value of c where the **instantaneous rate of change at $x = c$ is equal to the average rate of change of f on the interval $[a, b]$.**

AP Calculus AB **Name: ______________________________**
Unit 5 – Day 2 – Assignment

For the exercises 1 – 5, determine whether Rolle's Theorem can be applied to the function on the indicated interval. If Rolle's Theorem can be applied, find all values of *c* that satisfy the theorem.

1. $f(x) = x^2 - 4x$ on the interval $0 \le x \le 4$	2. $f(x) = (x+4)^2(x-3)$ on the interval $-4 \le x \le 3$
3. $f(x) = 4 - \lvert x-2 \rvert$ on the interval $-3 \le x \le 7$	4. $f(x) = \sin x$ on the interval $0 \le x \le 2\pi$
5. $f(x) = \cos 2x$ on the interval $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$	

For exercises 6 – 9, determine whether the Mean Value Theorem can be applied to the function on the indicated interval. If the Mean Value Theorem can be applied, find all values of *c* that satisfy the theorem.

6. $f(x) = x^3 - x^2 - 2x$ on $-1 \le x \le 1$	7. $f(x) = \sqrt{x-3}$ on $3 \le x \le 7$
8. $f(x) = \frac{x+2}{x}$ on $\frac{1}{2} \le x \le 2$	9. $h(x) = 2\cos x + \cos 2x$ on $0 \le x \le \pi$