AP Calculus AB
Unit 5 – Day 2 – Assignment

Name: Answer Key *

For the exercises 1 – 5, determine whether Rolle's Theorem can be applied to the function on the indicated interval. If Rolle's Theorem can be applied, find all values of c that satisfy the theorem.

1. $f(x) = x^2 - 4x$ on the interval $0 \le x \le 4$

$$\left.\begin{aligned}
&\bullet f(x) \text{ is cont on } [0,4] \checkmark \\
&\bullet f(x) \text{ is diff. on } (0,4) \checkmark \\
&\bullet f(0) = (0)^2 - 4(0) = 0 \\
& f(4) = (4)^2 - 4(4) = 0 \\
& f(0) = f(4) \checkmark
\end{aligned}\right\} \text{ Rolle's is applicable } \checkmark$$

$f'(x) = 2x - 4$

$f'(c) = 2c - 4 = 0$

$2c = 4$

$\boxed{c = 2}$

2. $f(x) = (x+4)^2(x-3)$ on the interval $-4 \le x \le 3$

$$\left.\begin{aligned}
&\bullet f(x) \text{ is cont on } [-4,3] \checkmark \\
&\bullet f(x) \text{ is diff on } (-4,3) \checkmark \\
&\bullet f(-4) = (-4+4)^2(-4-3) = 0 \\
& f(3) = (3+4)^2(3-3) = 0 \\
& f(-4) = f(3) \checkmark
\end{aligned}\right\} \text{ Rolle's applicable}$$

$f'(x) = 2(x+4)(1)(x-3) + (x+4)^2(1)$

$= (2x+8)(x-3) + (x+4)^2$

$= 2x^2 - 6x + 8x - 24 + x^2 + 8x + 16$

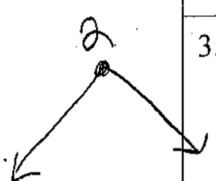
$= 3x^2 + 10x - 8$

$3c^2 + 10c - 8 = 0$

$(3c-2)(c+4)$

$\boxed{c = \tfrac{2}{3}}$ $\cancel{c = -4}$

3. $f(x) = 4 - |x-2|$ on the interval $-3 \le x \le 7$



$f(x)$ is not differentiable at $x = 2$, which is on $[-3, 7]$, since the graph of $f(x)$ has a cusp at $x = 2$.

$\therefore$ Rolle's Thm is not applicable

4. $f(x) = \sin x$ on the interval $0 \le x \le 2\pi$

$$\left.\begin{aligned}
&\bullet f(x) \text{ is cont on } [0, 2\pi] \checkmark \\
&\bullet f(x) \text{ is diff on } (0, 2\pi) \checkmark \\
&\bullet f(0) = \sin(0) = 0 \\
& f(2\pi) = \sin(2\pi) = 0 \\
& f(0) = f(2\pi) \checkmark
\end{aligned}\right\} \text{ Rolle's Thm applicable}$$

$f'(x) = \cos x$

$\cos(c) = 0$

$\boxed{c = \pi/2 \ \& \ c = 3\pi/2}$

5. $f(x) = \cos 2x$ on the interval $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$

$$\left.\begin{aligned}
&\bullet f(x) \text{ is cont on } [\pi/3, 2\pi/3] \checkmark \\
&\bullet f(x) \text{ is diff on } (\pi/3, 2\pi/3) \checkmark \\
&\bullet f(\pi/3) = \cos\left(2\left(\tfrac{\pi}{3}\right)\right) = -1/2 \\
& f(2\pi/3) = \cos\left(2\left(\tfrac{2\pi}{3}\right)\right) = -1/2 \\
& f(\pi/3) = f(2\pi/3) \checkmark
\end{aligned}\right\} \text{ Rolle's applicable}$$

$f'(x) = -2\sin 2x$

$-2\sin(2c) = 0$

$\sin(2c) = 0$

$2c = \pi, 2\pi, 3\pi, 4\pi, \ldots$

$2c = \pi$

$\boxed{c = \pi/2}$, $\cancel{\frac{2\pi}{2}}$, $\cancel{\frac{3\pi}{2}}$, $\cancel{\frac{4\pi}{2}}$

For exercises 6 – 9, determine whether the Mean Value Theorem can be applied to the function on the indicated interval. If the Mean Value Theorem can be applied, find all values of c that satisfy the theorem.

6. $f(x) = x^3 - x^2 - 2x$ on $-1 \le x \le 1$

- f(x) is cont. on [-1, 1] ✓
- f(x) is diff on (-1, 1) ✓

m.V.T applicable

$f(-1) = (-1)^3 - (-1)^2 - 2(-1)$

$f(1) = (1)^3 - (1)^2 - 2(1)$

$f'(x) = 3x^2 - 2x - 2$

$3c^2 - 2c - 2 = \frac{f(-1) - f(1)}{-1-1}$

$3c^2 - 2c - 2 = \frac{0+2}{-2}$

$3c^2 - 2c - 2 = -1$

$3c^2 - 2c - 1 = 0$

$(3c^2 - 3c) + (c - 1)$

$3c(c-1) + 1(c-1)$

$(3c+1)(c-1) = 0$

$\boxed{c = -1/3}$ $\quad \cancel{c = 1}$

7. $f(x) = \sqrt{x-3}$ on $3 \le x \le 7$

$x - 3 \ge 0$

$x \ge 3$

Although f(x) is defined at x = 3, f(x) is not continuous at x = 3 b/c the domain is $[3, \infty)$.

∴ Mean Value Theorem is not applicable.

8. $f(x) = \frac{x+2}{x}$ on $\frac{1}{2} \le x \le 2$

$x \ne 0$

- f(x) is cont. on [1/2, 2] ✓
- f(x) is diff on (1/2, 2) ✓

m.V.T applicable

$f'(x) = \frac{(x)(1) - (x+2)(1)}{(x^2)}$

$= \frac{x - x - 2}{x^2}$

$f(\tfrac{1}{2}) = \frac{\frac{1}{2}+2}{\frac{1}{2}} = 5$

$f(2) = \frac{2+2}{2} = 2$

$\Rightarrow \frac{-2}{c^2} = \frac{f(\frac{1}{2}) - f(2)}{\frac{1}{2} - 2}$

$\frac{-2}{c^2} = \frac{5-2}{\frac{1}{2}-2}$

$\frac{-2}{c^2} = \frac{3}{-1.5}$

$3c^2 \quad 3$

$c^2 = 1$

$c = \pm 1$

$\boxed{c = 1}$

9. $h(x) = 2\cos x + \cos 2x$ on $0 \le x \le \pi$

- h(x) is cont on $[0, \pi]$ ✓
- h(x) is diff on $(0, \pi)$ ✓

m.V.T applicable

$h'(x) = -2\sin x + -2\sin 2x$

$-2\sin c - 2\sin 2c = \frac{f(0) - f(\pi)}{0 - \pi}$

$f(0) = 2\cos 0 + \cos 2 0 = 3$

$f(\pi) = 2\cos\pi + \cos(2\pi) = -1$

$-2\sin c - 2\sin 2c = \frac{3-(-1)}{0-\pi}$

$-2\sin c - 2\sin 2c = -1.273$

$\boxed{c = 0.217 \text{ \& } c = 1.748}$