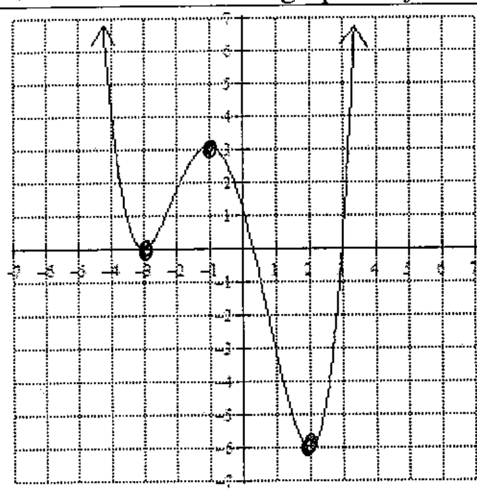
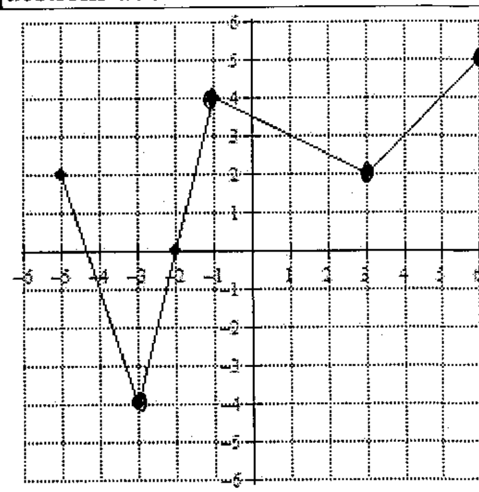
AP Calculus
Unit 5 – Applications of the Derivative – Part 2

Day 1 Notes: The Extreme Value Theorem

Definitions of Relative and Absolute Extrema of a Function

- Relative Extrema are points where a function changes from increasing to decreasing or vice versa.
- Absolute Extrema are the point(s) on a graph with the greatest and/or least y-value(s).

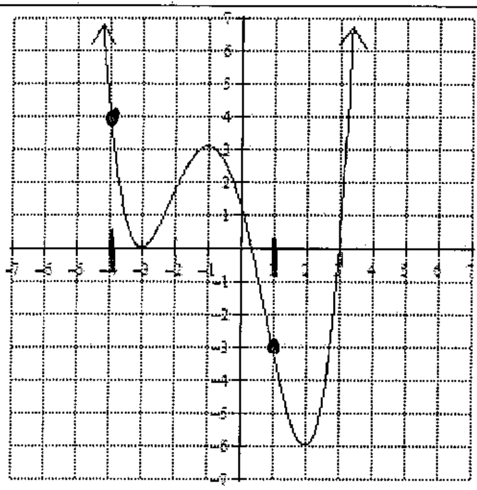
Pictured below are the graphs of f and g. Answer the questions about these two functions.

 Graph of $f(x)$	 Graph of $g(x)$
Identify the coordinates of the relative extrema of f. rel. min → (-3, 0) & (2, -6) rel. max → (-1, 3)	Identify the coordinates of the relative extrema of g. rel. min → (-3, -4) & (3, 2) rel. max → (-1, 4)
On the domain of f, what are the coordinates of the absolute extrema of f? abs min → (2, -6) abs max → none	On the domain of g, what are the coordinates of the absolute extrema of g? abs min → (-3, -4) abs max → (6, 5)

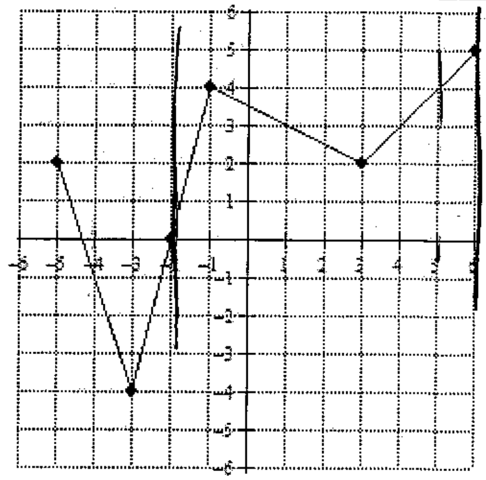
On the domain of the given function, did the absolute extrema occur at the function's relative extrema?

- f(x) → ABS. min occured at rel. min
- g(x) → ABS. min occured at rel. min

ABS. max did not occur at a rel. max.



Graph of $f(x)$



Graph of $g(x)$

On the interval $-2 \le x \le 3$, what are the absolute extrema of f?

abs min → (2, -6)

abs max → (-1, 3)

On the interval $-4 \le x \le 5$, what are the absolute extrema of g?

abs min → (-3, -4)

abs max → (-1, 4) & (5, 4)

On the interval $-4 \le x \le 1$, what are the absolute extrema of f?

abs min → (-4, 4)

abs max → (1, -3)

On the interval $-2 \le x \le 6$, what are the absolute extrema of g?

abs min → (-2, 0)

abs max → (6, 5)

When the domain is restricted to a particular closed interval, at what three places ~~that~~ can the absolute extrema ~~could~~ exist?

*

1. At the endpoints of the closed interval.
2. At any x-value where $f'(x) = 0$
3. At any x-value where $f'(x)$ is undefined

The Extreme Value Theorem (E. V. T.):

If $f(x)$ is continuous on $[a,b]$, then there exists an absolute maximum and an absolute minimum on $[a,b]$ at either $x=a$, $x=b$, or any value of x on (a,b) such that $f'(x) = 0$ or $f'(x)$ is undefined.

Use the extreme value theorem to locate the absolute extrema of the function $f(x) = -x^3 - 6x^2 - 9x + 2$ on the given closed intervals. ~~Your algebraic results should concur with your graphical conclusions from the previous page.~~

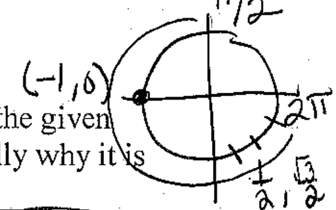
Interval: $-4 \le x \le -1$	Interval: $-4 \le x \le 1$
$f'(x) = -3x^2 - 12x - 9$ $-3(x^2 + 4x + 3) = 0$ $-3(x+3)(x+1) = 0$ $x = -3 \quad x = -1$ endpoints: $f(-4) = 6$, $f(-1) = 6$ $f'(x) = 0$: $f(-3) = 2$ • abs min → $(-3, 2)$ • abs max → $(-4, 6)$ & $(-1, 6)$	endpts: $f(-4) = 6$, $f(1) = -14$ $f'(x) = 0$: $f(-3) = 2$, $f(-1) = 6$ • abs min → $(1, -14)$ • abs max → $(-4, 6)$ & $(-1, 6)$

For each of the following functions, state specifically why the E. V. T. is or is not applicable on the given interval.

Interval: $-5 \le x \le 0$	
$H(x) = \frac{3x+2}{x+3}$ $x \neq -3$	The E.V.T is not applicable on $-5 \le x \le 0$ for $H(x)$ b/c $H(x)$ is discontinuous at $x = -3$.
$G(x) = 2x\sqrt{x-3}$ $x - 3 \ge 0$ $x \ge 3$	The E.V.T. is not applicable on $-5 \le x \le 0$ for $G(x)$ b/c the domain of $G(x)$ is $(3, \infty)$ which means $G(x)$ is not defined → for all of $[-5, 0]$
$f(x) = \ln(x+7)$ $x + 7 > 0$ $x > -7$	The E.V.T is applicable on $-5 \le x \le 0$ for $f(x)$ b/c the domain for $f(x)$ is $(-7, \infty)$ So $f(x)$ is defined for all of $[-5, 0]$

$\frac{-9}{1}$

CALC. ACTIVE



Given the functions below, determine the absolute extreme values of the function on the given interval, provided the extreme value theorem is applicable. If it is not, state specifically why it is not.

E.V.T. applicable ✓

1. $f(x) = x^3 - 2x^2 - 3x - 2$ on $[-1, 3]$

$f'(x) = 3x^2 - 4x - 3$

$3x^2 - 4x - 3 = 0$

$x = -.535 \quad x = 1.869$ when $f'(x) = 0$

$f(-1) = -2$

$f(3) = -2$

$f(-.535) = -1.121$

$f(1.869) = -8.065$

* abs min → $(1.869, -8.065)$

* abs max → $(-.535, -1.121)$

2. $g(x) = \sin^2 x + \cos x$ on $\frac{\pi}{2} \le x \le 2\pi$

$(\sin x)^2 + \cos x$

$g'(x) = 2(\sin x)'(\cos x) + -\sin x$

$= 2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad 2\cos x = 1$

$x = \pi \quad \cos x = 1/2$

$x = \frac{5\pi}{3}$

$g(\pi/2) = 1$

$g(2\pi) = 1$

$g(\pi) = -1$

$g(\frac{5\pi}{3}) = 1.25$

* abs min → $(\pi, -1)$

* abs max → $(\frac{5\pi}{3}, 1.25)$

3. $f(x) = (x+2)^{\frac{2}{3}}$ on $[-3, 6]$

$f'(x) = \frac{2}{3}(x+2)^{-1/3}(1)$

$\frac{2}{3\sqrt[3]{x+2}}$

$f'(x) \neq 0$, but $f'(x)$ is undefined at $x = -2$

$f(-3) = 1$

$f(6) = 4$

$f(-2) = 0$

* abs min → $(-2, 0)$

* abs max → $(6, 4)$

4. $h(x) = \ln(x^2 - 4)$ on $[-1, 3]$

$x^2 - 4 > 0$

$x^2 > 4$

$x > 2$

The E.V.T is not applicabe b/c at $x = 2$, which is on $[-1, 3]$, $h(x)$ is undefined, and therefore, not continuous.