

- 1) If $f(x) = \sqrt{3x+1}$, what is the slope of the normal line to the graph of $f(x)$ when $x = 1$?

$$f(x) = (3x+1)^{1/2}$$

$$f'(x) = \frac{1}{2}(3x+1)^{-1/2} (3) = \frac{3}{2\sqrt{3(1)+1}} = \frac{3}{2\sqrt{4}} = \frac{3}{4} \leftarrow \text{slope of tangent}$$

Slope of normal = $-4/3$

- 2) Find $\lim_{h \rightarrow 0} \frac{\csc 5(x+h) - \csc 5x}{h}$

$$f(x) = \csc 5x$$

$$f'(x) = -\csc 5x \cot 5x (5) = \boxed{-5 \csc 5x \cot 5x}$$

- 3) Is the graph of $f(x) = \frac{x}{x-2}$ increasing or decreasing at $x = 3$?

$$f'(x) = \frac{(x-2)(1) - (x)(1)}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$$f'(3) = \frac{-2}{(3-2)^2} = \frac{-2}{1} = -2$$

Since $f'(3) < 0$, $f(x)$ is decreasing at $x=3$.

- 4) Find the equation of the tangent line to the graph of $f(x) = x^2 \cos x$ when $x = \pi$.

$$f(\pi) = \pi^2 (\cos(\pi)) = \pi^2 (-1) = -\pi^2 \quad \text{P.O.T} = (\pi, -\pi^2)$$

$$f'(x) = (2x)(\cos x) + (x^2)(-\sin x)$$

$y + \pi^2 = -2\pi(x - \pi)$

$$f'(\pi) = (2\pi)(\cos \pi) + (\pi^2)(-\sin \pi) = 2\pi(-1) + \pi^2(0) = -2\pi = \text{S.O.T}$$

#s 5 - 8: Find the derivative of each function.

5) $f(x) = 4x\sqrt{x+2}$

$$f(x) = 4x(x+2)^{1/2}$$

$$f'(x) = (4)(x+2)^{1/2} + (4x) \frac{1}{2}(x+2)^{-1/2} (1)$$

$$\frac{(\sqrt{x+2}) 4\sqrt{x+2} + \frac{2x}{\sqrt{x+2}}}{(\sqrt{x+2})} = \frac{4(x+2) + x}{\sqrt{x+2}} = \frac{4x+8+x}{\sqrt{x+2}}$$

$\frac{5x+8}{\sqrt{x+2}}$

$$6) f(x) = \cos\left(\frac{4x+1}{x-1}\right)$$

$$f'(x) = -\sin\left(\frac{4x+1}{x-1}\right) \left(\frac{(x-1)(4) - (4x+1)(1)}{(x-1)^2} \right)$$

$$\left(\frac{4x-4-4x-1}{(x-1)^2} \right)$$

$$-\sin\left(\frac{4x+1}{x-1}\right) \left(\frac{-5}{(x-1)^2} \right) = \boxed{\frac{5}{(x-1)^2} \sin\left(\frac{4x+1}{x-1}\right)}$$

$$7) f(x) = \frac{\sqrt{6x+1}}{x-2}$$

$$f(x) = \frac{(6x+1)^{1/2}}{x-2}$$

$$f'(x) = \frac{(x-2)^{1/2}(6x+1)^{-1/2}(6) - (6x+1)^{1/2}(1)}{(x-2)^2}$$

$$\frac{3(x-2)}{\sqrt{6x+1}} - \frac{\sqrt{6x+1} \cdot \sqrt{6x+1}}{(\sqrt{6x+1})^2}$$

$$\frac{3x-6-6x-1}{\sqrt{6x+1}} = \frac{-3x-7}{\sqrt{6x+1}}$$

$$\boxed{\frac{-3x-7}{(x-2)^2 \sqrt{6x+1}}}$$

$$8) f(x) = \frac{e^{4x}}{x+1}$$

$$f'(x) = \frac{(x+1)(e^{4x})(4) - (e^{4x})(1)}{(x+1)^2}$$

$$= \frac{4xe^{4x} + 4e^{4x} - e^{4x}}{(x+1)^2} = \frac{4xe^{4x} + 3e^{4x}}{(x+1)^2} = \boxed{\frac{e^{4x}(4x+3)}{(x+1)^2}}$$