AP Calculus
Unit 3 – Rules of Differentiation

Day 6 Notes: Derivatives of Inverse Functions

Given a function, $f(x)$, the inverse function, $f^{-1}(x)$, is numerically defined to be

_____.

Graphical Representation of the Inverse	**Analytical Representation of the Inverse**

Consider the two functions, $f(x)$ and $g(x)$, represented numerically below. Answer the questions that follow.

x	$f(x)$	$g(x)$
-2	3	1
1	2	-2

Complete the table of values below.

x	$f^{-1}(x)$

Complete the table of values below.

x	$g^{-1}(x)$

Find the value of $f\left(g^{-1}(-2)\right)$.

Find the value of $f^{-1}\left(f(1)\right)$.

Find the value of $g^{-1}\left(f^{-1}(2)\right)$.

Find the value of $f^{-1}\left(g\left(g^{-1}(1)\right)\right)$.

Finding a Formula for the Derivative of an Inverse

Differentiate both sides of the equation below.

$$f[f^{-1}(x)] = x$$

Suppose that $f(x) = 3x + 2$ and $f'(-2) = 3$. What is the value of $[f^{-1}(-4)]'$?

Given to the right is a table of values for f, g, f', and g'. Use the values in the table to find each indicated value in the boxes below.

x	f	g	f'	g'
−2	1	2	0	3
0	−4	−3	−1	2
1	3	−2	2	1
3	1	1	−3	−2

Find the value of $[f^{-1}(3)]'$.

Find the value of $[g^{-1}(-2)]'$.

Find the value of $[g^{-1}(1)]'$. Then, find the equation of the line tangent to the graph of g^{-1} when $x = 1$.

Estimate the value of $f'(2)$. Based on this value, what conclusion can be reached about the graph of f when $x = 2$? Explain your reasoning.

AP Calculus AB **Name:** _____
Unit 3 – Day 6 – Assignment

MULTIPLE CHOICE

1. Which of the following statements can be made about the graph of the function $h(x) = \dfrac{\ln(\cos x)}{\tan x}$ when $x = \dfrac{\pi}{2}$.

A. The graph of $h(x)$ is increasing.

B. The graph of $h(x)$ is decreasing.

C. No conclusion can be made about the graph of $h(x)$.

D. The graph of $h(x)$ has a horizontal tangent.

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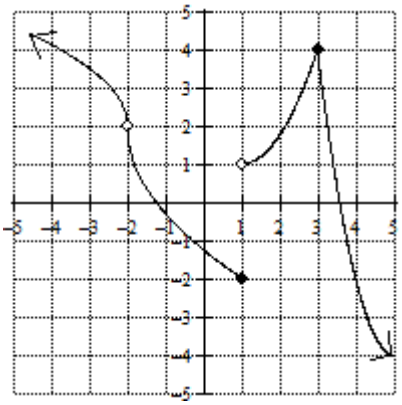
2. Consider the graph of $f(x)$ to the right to determine which of the following statements is/are true.

I. $f'(x) = 0$ when $x = 3$.

II. $f'(2.5) > 0$.

III. On the interval $(-4,5)$ there are three values of x at which $f(x)$ is not differentiable.



A. II and III only
B. I and II only
C. III only
D. I, II and III

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3. Let $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$ and $g'(7) = \dfrac{1}{7}$. Find $h'(7)$ if $h(x) = \dfrac{f(x)}{g(x)}$.

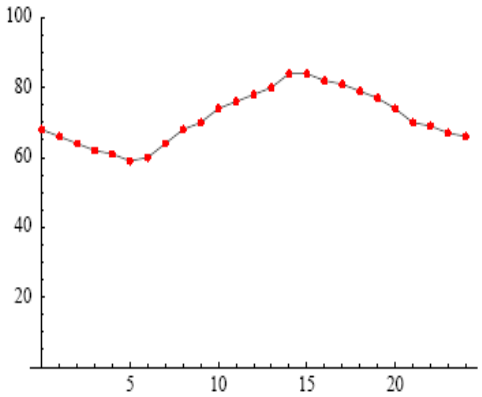
A. 98

B. –14

C. –2

D. 14

4. The graph to the right shows data of a function, $H(t)$, which shows the relationship between temperature in °C (y-axis) and the time in hours (x-axis). What does the value of $H'(6)$ represent?



A. $H'(6)$ represents the temperature after 6 hours measured in °C.
B. $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C.
C. $H'(6)$ represents the temperature after 6 hours measured in °C per hour
D. $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C per hour.

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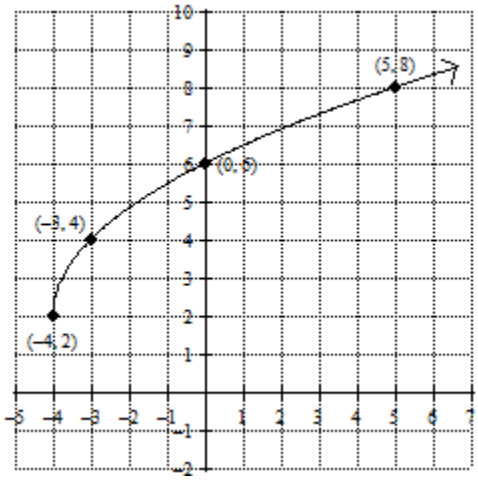
5. The graph of $h(x) = 2\sqrt{x+4}+2$ is pictured below. What is the value of $[h^{-1}(6)]'$?



A. $\frac{1}{4}$
B. 2
C. 4
D. $\frac{1}{2}$

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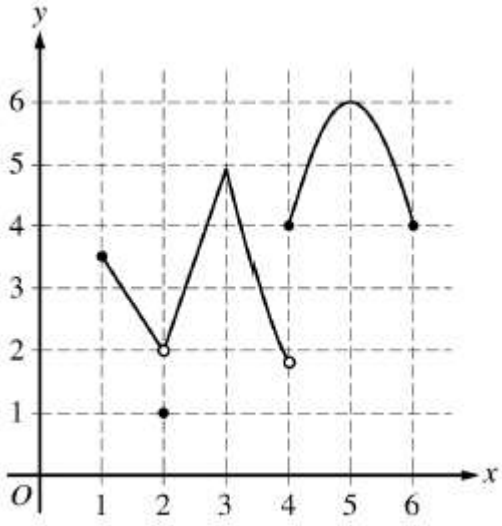
6. Find y' if $y = x^2e^x$.

A. $2xe^x$
B. $x\left(x+2e^x\right)$
C. $xe^x(x+2)$
D. $2x+e^x$

7. The function f is pictured to the right. At which of the following values of x is f defined and continuous but not differentiable.

I. $x = 2$ II. $x = 3$ III. $x = 5$

A. II only
B. I only
C. II and III only
D. I and II only



Graph of f

FREE RESPONSE

The table below shows values of differentiable functions, $f(x)$ and $g(x)$, and their derivatives at selected values of x. Use the table of values below to answer each of the questions below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	−1	2	5
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

a. Approximate the value of $f'(1.5)$? Explain why your answer is a good approximation of $f'(1.5)$.

b. If $B(x) = \sqrt{g(x)}$, what is the equation of the tangent line drawn to $B(x)$ when $x = 1$?

c. If $A(x) = x^2 \ln(f(x))$, what is the value of $A'(2)$? What does this result say about the behavior of the graph of $A(x)$ when $x = 2$? Give a reason for your answer.

d. Find the value of $[g^{-1}(3)]'$. Then, find the equation of the line normal to the graph of $g^{-1}(x)$ at $x = 3$.