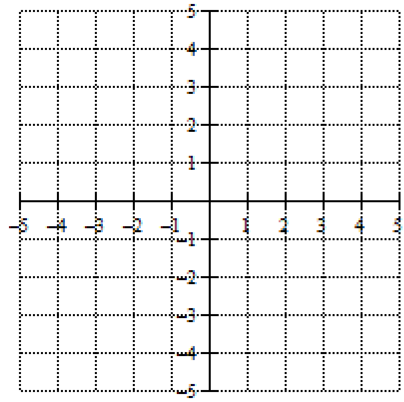
AP Calculus
Unit 3 – Rules of Differentiation

Day 5 Notes: The Relationship Between Continuity & Differentiability

Example 1:

Consider the function $f(x)=\sqrt{x^2-4}$ at $x = 2$.

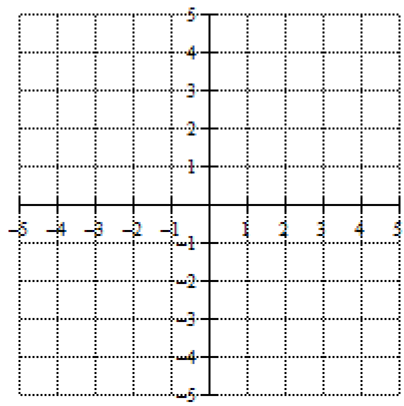


Based on the graph, if $f(x)$ continuous at $x = 2$? Explain your reasoning.

Find the value of $f\,'(2)$ to determine if $f(x)$ is differentiable at $x = 2$.

Example 2:

Consider the function $f(x)=x^{\frac{1}{3}}+2$ at $x = 0$. .

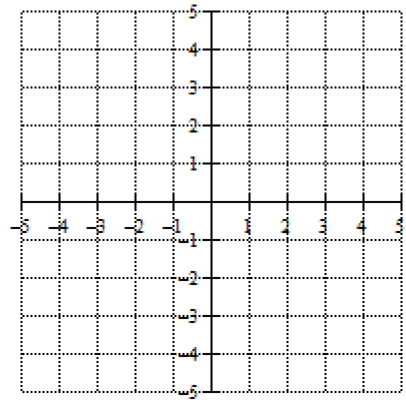


Based on the graph, if $f(x)$ continuous at $x = 0$? Explain your reasoning.

Find the value of $f\,'(0)$ to determine if $f(x)$ is differentiable at $x = 0$.

<u>Example 3</u>:

Consider the function $f(x) = x^{\frac{2}{3}} + 2$ at $x = 0$.



Based on the graph, if $f(x)$ continuous at $x = 0$? Explain your reasoning.

Find the value of $f\,'(0)$ to determine if $f(x)$ is differentiable at $x = 0$.

<u>Graphically</u>, a function is not differentiable if….

1)

2)

3)

In order for a function to be differentiable at a value of x, then two things must be true:

1)

2)

Example 4:

Consider the function $g(x)=\begin{cases}\sqrt{x+1}, & 0\le x\le 3\\ 5-x, & 3<x\le 5\end{cases}$ to answer the following questions.

Is $g(x)$ continuous at $x = 3$? Show the complete analysis.

Is $g(x)$ differentiable at $x = 3$? Show the complete analysis.

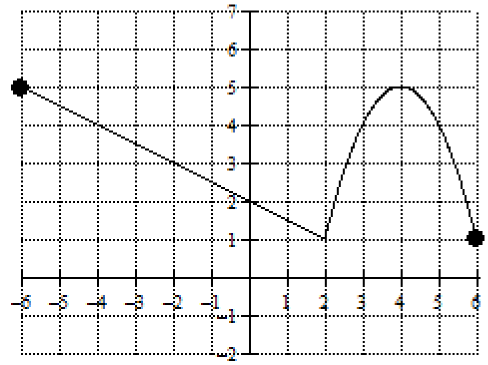
Example 5:

For what values of k and m will the function below be both continuous and differentiable at $x = 3$?

$$h(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2, & 3<x\le 5\end{cases}$$

AP Calculus AB **Name: ______________________________**
Unit 3 – Day 5 – Assignment

Use the graph of $H(x)$, pictured to the right, to complete exercises 1 – 4.



1. The graph of $H(x)$ is continuous on its domain but not differentiable at all values on its domain. At what value on $-6 < x < 6$ is $H(x)$ not differentiable. Give a graphical reason for your answer.

2. Write an equation of $H(x)$ and show analytically that $H(x)$ is, in fact, continuous at the x – value that you identified in exercise 1. Show and explain your work.

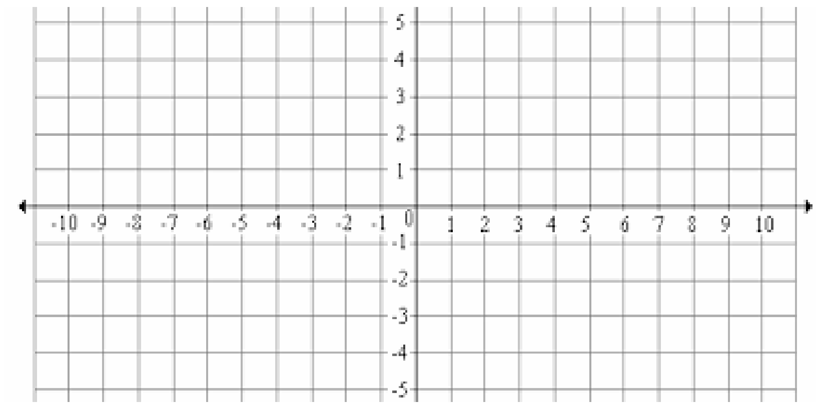
3. Show analytically that $H(x)$ is, in fact, not differentiable at the x – value that you identified in exercise 1. Show and explain your work.

4. Given the graph of $H(x)$ pictured above, find the equation of the tangent line to the graph of $P(x) = \sqrt{H(x)}$ when $x = 3$.

A continuous function on the interval $-4 < x < 5$, $h(x)$, is described in the table below. Use the information to complete exercises 5 – 8.

x	–4	– 2	–1	0	$-4 < x < 0$	1	3	$0 < x < 3$	$3 < x < 5$	5
h(x)	–5	–4	–2	1	Increasing & Concave Up	–1	–2	Decreasing & Concave Up	Increasing & Concave Up	0

5. Sketch a graph of $h(x)$.



6. Estimate the value of $h'(-2)$. Does this value support the claim that $h(x)$ is increasing on the interval $-4 < x < 0$? Give a reason for your answer.

7. There are three x – values in the domain of h at which $h(x)$ is not differentiable. What are these three values and give a reason for why $h(x)$ is not differentiable at these values.

8. On what interval(s) of x is $h'(x) > 0$? Give a reason for your answer.

9. At what value(s) of x will the graph of $f(x) = 2e^{2x} - 3x$ have a tangent line whose slope is 1?

10. The graph of $x - 2y = 9$ is parallel to the normal line to the graph of $f(x)$ when $x = 5$. What is the value of $f'(5)$? Justify your answer.

11. Let f be defined by the function $f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$.

a. If the function is continuous at $x = 1$, what is the relationship between a and b? Explain your reasoning using limits.

b. Find the unique values of a and b that will make f both continuous and differentiable at $x = 1$. Show your analysis using limits.

12. For what values of a and b will the function below be differentiable at $x = 1$?

$$f(x) = \begin{cases} 3ax^2 + 2bx + 1, & x \leq 1 \\ ax^4 - 4bx^2 - 3x, & x > 1 \end{cases}$$