AP Calculus
Unit 3 – Rules of Differentiation

Day 4 Notes: Finding the Derivative of the Natural Exponential & Logarithmic Functions

Differentiation Rule for Natural Exponential Functions

Find the derivative of each of the following functions.

$f(x) = e^{\sin x}$	$f(x) = e^{2x+3}$
$f(x) = 3e^{2x}$	$f(x) = (2x+3)e^{3x}$
$f(x) = x^2 e^{2x}$	$f(x) = \sqrt{e^{2x-6}}$

$$f(x) = \frac{e^{5x}}{3x^2}$$

Differentiation Rule for Natural Logarithmic Functions

Find the derivative of each of the following functions.

$f(x) = \ln(2x - 3)$	$f(x) = \ln\left(3x^2 + 2x\right)$
$f(x) = \ln(\cos x)$	$f(x) = \ln\sqrt{2x - 4}$

Finding Values of Derivatives Using the Graphing Calculator

For each of the functions below, find the value of $f'(x)$ at the indicated value of x using the graphing calculator. Then, determine if the function is increasing, decreasing, has a horizontal tangent or has a vertical tangent. Give a reason for your answer.

Function	Value of $f'(a)$	Is $f(x)$ increasing or decreasing, or does $f(x)$ have a horizontal or a vertical tangent?
1. $f(x) = 3e^x \sin x$	$a = -2$	
2. $f(x) = 3e^x \sin x$	$a = 1$	
3. $f(x) = \frac{\ln(\cos x)}{x^2}$	$a = \frac{\pi}{3}$	
4. $f(x) = \frac{\ln(\cos x)}{x^2}$	$a = \pi$	
5. $f(x) = e^{\tan(0.34x)}$	$a = 0$	
6. $f(x) = 5\sin^2(\ln x)$	$a = 1$	

We already understand the derivative to be the **SLOPE OF THE TANGENT LINE**. Slope is a **rate**. Therefore, the derivative of a function actually represents the **RATE AT WHICH A FUNCTION IS CHANGING.**

7.	The number of people entering a concert can be modeled by the function $f(t) = 560e^{\sin t}$, where *t* represents the number of hours after the gates are open.
a.	Find the values of $f\left(\frac{1}{2}\right)$ and $f'\left(\frac{1}{2}\right)$. Using correct units, explain what each value represents in the context of this problem.
b.	How many people have entered the concert 2 hours after the gates are opened? Is the number of people entering increasing or decreasing at this time? Justify your answer.

8.	After being poured into a cup, coffee cools so that its temperature, *T*(*t*), is represented by the function $T(t) = 70 + 110e^{-t/2}$, where *t* is measured in minutes and *T*(*t*) is measured in degrees Fahrenheit.
a.	What is the temperature of the coffee 5 minutes after it has been poured into the cup?
b.	Is the temperature decreasing faster 1 minute after it is poured or 3 minutes after it is poured? Give a reason for your answer.

AP Calculus AB **Name: ______________________________**
Unit 3 – Day 4 – Assignment

In exercises 1 – 10, find the derivative of the function. Express your answer in simplest factored form.

1. $F(x) = x^3 e^{2x}$	2. $P(x) = e^{-2x^2}$
3. $H(x) = e^{x \ln x}$	4. $g(x) = \left(2x^2 + 3\right)e^x$
5. $J(x) = \ln\left(e^{2x} + 1\right)$	6. $F(x) = \ln(3 - 2x)$

7. $K(x) = \ln\sqrt{5x-2}$	8. $F(x) = x^2 e^{4x}$
9. $T(x) = \dfrac{\ln x}{x-2}$	10. $P(x) = \dfrac{e^{2x}}{x^3}$

11. Find the equation of the tangent line to the graph of $y = \dfrac{\ln x}{4x}$ when $x = 1$.