* Unit 3 - Day 3 - Scavenger Hunt *

A.

Find f'(x) of $f(x) = (2x + 3)^3$

$$\frac{3}{\sqrt{8}}$$

$f(x) = (2x+3)^3$

$f'(x) = 3(2x+3)^2(2)$

$= 6(2x+3)^2$

$= 6(4x^2+12x+9)$

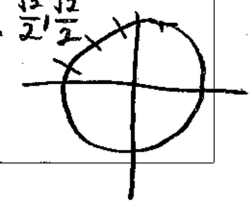
$= \boxed{24x^2+72x+54}$

S.

Find the slope of the normal line to the graph of $f(\theta) = \sin^2\theta$ when $\theta = \frac{3\pi}{4}$

$$24x^2 + 72x + 54$$

$-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$



$f(\theta) = (\sin\theta)^2$

$f'(\theta) = 2(\sin\theta)'(\cos\theta)$

$f'(\frac{3\pi}{4}) = 2(\sin\frac{3\pi}{4})(\cos\frac{3\pi}{4})$

$= 2(\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2}) = -\frac{2(2)}{4} = -1$

S.O.T

$\boxed{\text{slope of normal} = 1}$

R.

Find g'(x) if $g(x) = \sqrt{2x + 5}$

$$1$$

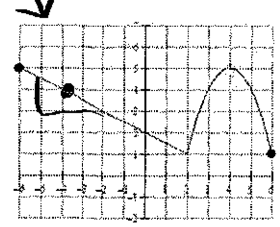
$g(x) = (2x+5)^{1/2}$

$g'(x) = \cancel{\frac{1}{2}}(2x+5)^{-1/2}(\cancel{2})$

$= \boxed{\frac{1}{\sqrt{2x+5}}}$

I.

Given the graph of H(x), find the equation of the tangent line to the graph of $P(x) = \sqrt{H(x)}$ when x = -4.



$$\frac{1}{\sqrt{2x + 5}}$$

$\boxed{y - 2 = -\frac{1}{8}(x+4)}$

• P.O.T → $P(-4) = \sqrt{H(-4)} = \sqrt{4} = 2$ $(-4, 2)$

• S.O.T → $P(x) = [H(x)]^{1/2}$

$P'(x) = \frac{1}{2}[H(x)]^{-1/2}[H'(x)]$

$-\frac{1}{8}$

$P'(-4) = \frac{1}{2}[H(-4)]^{-1/2}[H'(-4)]$

$\frac{1}{2}(4)^{-1/2}(-\frac{1}{2}) = \frac{1}{2}(\frac{1}{\sqrt{4}})(\frac{-1}{2}) = \frac{-1}{4}(\frac{1}{2})$

B.

Find the derivative of
$G(x) = \cos^2 3x$

$y - 2 = -1/8(x + 4)$

$G(x) = [\cos(3x)]^2$

$G'(x) = 2[\cos(3x)]^1(-\sin(3x))(3)$

$\boxed{-6\cos 3x \sin 3x}$

Q.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	2	−1	0	−1
3	−5	4	−4	6
4	1	7	8	−2

Is the graph of $h(x) = f(g(x))$ increasing, decreasing, or at a relative maximum or minimum when $x = 3$?

$-6\cos 3x\sin 3x$

$h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(3) = f'(g(3)) \cdot g'(3)$

$f'(4) \cdot (6)$

$8 \cdot 6 = 48$

since $h'(3) > 0$,
$h(x)$ is $\boxed{\text{increasing}}$

J.

Find the derivative of

$f(x) = \left(\dfrac{x + 5}{x^2 + 2}\right)^3$

increasing x^2+2-2x^2-10x

$f'(x) = 3\left(\dfrac{x+5}{x^2+2}\right)^2\left(\dfrac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}\right)$

$3\left(\dfrac{x+5}{x^2+2}\right)^2\left(\dfrac{-x^2-10x+2}{(x^2+2)^2}\right)$

$\boxed{\dfrac{-3(x+5)^2(x^2+10x-2)}{(x^2+2)^4}}$

H.

If $f(\theta) = \csc 2\theta$, then $f'(x) =$ ___?

$-\csc 2\theta \cot 2\theta\,(2)$

$\boxed{-2\csc 2\theta \cot 2\theta}$

$\dfrac{-3(x + 5)^2(x^2 + 10x - 2)}{(x^2 + 2)^4}$

P.

Find h'(x) if
$h(x) = \sqrt{x^2 - 3x + 1}$

$-2\csc 2\theta \cot 2\theta$

$h(x) = (x^2 - 3x + 1)^{1/2}$

$h'(x) = \frac{1}{2}(x^2 - 3x + 1)^{-1/2}(2x - 3)$

$(x - \frac{3}{2})(x^2 - 3x + 1)^{-1/2}$

$\frac{x - \frac{3}{2} \quad (2)}{\sqrt{x^2 - 3x + 1}\,(2)} = \boxed{\frac{2x-3}{2\sqrt{x^2-3x+1}}}$

C.

If $f(x) = \sqrt{25 - x^2}$, find the equation of the normal line to the graph of f(x) when x = 3.

$$\frac{2x - 3}{2\sqrt{x^2 - 3x + 1}}$$

$f(3) = \sqrt{25 - (3)^2} = \sqrt{16} = 4$

$(3, 4)$

$f(x) = (25 - x^2)^{1/2}$

$f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x)$

$\frac{-x}{\sqrt{25 - x^2}}$

$\boxed{y - 4 = \frac{4}{3}(x - 3)}$

$f'(3) = \frac{-3}{\sqrt{25 - (-3)^2}} = \frac{-3}{4}$ S.O.N = 4/3

K.

Find the limit:

$$\lim_{h \to 0} \frac{\cos 3(x + h) - \cos 3x}{h}$$

$y - 4 = 4/3(x - 3)$

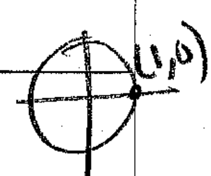
$f(x) = \cos 3x$

$f'(x) = -\sin(3x) \cdot (3)$

$= \boxed{-3\sin 3x}$

O.

Find g'(π) when
$g(\theta) = 1/4\sin^2 2\theta$



$-3\sin 3x$

$g(\theta) = \frac{1}{4}[\sin(2\theta)]^2$

$g'(\theta) = \frac{1}{2}[\sin(2\theta)]^1(\cos(2\theta))(2)$

$\frac{1}{2}\sin(2\pi)(\cos 2\pi)(2)$

$\frac{1}{2}(0)(1)(2) = \boxed{0}$

G.

Find f'(x) when
$f(x) = x\sqrt{1 - x^2}$

0

$f(x) = x\,(1-x^2)^{1/2}$

Product rule

$f'(x) = (1)(1-x^2)^{1/2} + (x)(\tfrac{1}{2}(1-x^2)^{-1/2}(-2x))$

$\frac{(\sqrt{1-x^2})}{(\sqrt{1-x^2})}\sqrt{1-x^2} + \frac{-x^2}{\sqrt{1-x^2}}$

$\frac{1-x^2 + -x^2}{\sqrt{1-x^2}} \boxed{= \frac{1-2x^2}{\sqrt{1-x^2}}}$

D.

If f(x) = tan4x, what is f'(x)?

$$\frac{\mathbf{1 - 2x^2}}{\mathbf{\sqrt{1 - x^2}}}$$

$f(x) = \tan 4x$

$f'(x) = \sec^2(4x)\cdot(4)$

$4\sec^2 4x$

L.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	2	−1	0	−1
3	−5	4	−4	6
4	1	7	8	−2

If p(x) = g(2x), what is the value of p'(1)?

$4\sec^2 4x$

$p(x) = g(2x)$

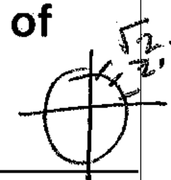
$p'(x) = g'(2x)\cdot(2)$

$p'(1) = g'(2(1))\cdot 2$

$g'(2)\cdot 2 = (-1)(2) = \boxed{-2}$

F.

Find the equation of the normal line to the graph of h(x) = tan(3x) when x = π/12?



-2

$(\frac{\pi}{12}, 1)$

$h(\frac{\pi}{12}) = \tan(3(\frac{\pi}{12})) = \tan(\frac{\pi}{4}) = 1$

$h'(x) = \sec^2(3x)\cdot 3$

$h'(\frac{\pi}{12}) = \sec^2(3(\frac{\pi}{12}))\cdot 3$

$= \sec^2(\frac{\pi}{4})\cdot 3$

$= (\frac{2}{\sqrt{2}})^2\cdot 3 = \frac{4}{2}(3) = 6 \leftarrow$ S.OT

$\boxed{\text{normal} = y - 1 = \frac{-1}{6}(x - \frac{\pi}{12})}$

M.

Find the derivative of

$$f(x) = \sqrt{\frac{2x + 3}{x - 2}}$$

$$y - 1 = -1/6(x - \pi/12)$$

$$f(x) = \left(\frac{2x+3}{x-2}\right)^{1/2}$$

$\cancel{2x-4} - 2x - 3$

$$f'(x) = \frac{1}{2}\left(\frac{2x+3}{x-2}\right)^{-1/2}\left(\frac{(x-2)(2) - (2x+3)(1)}{(x-2)^2}\right)$$

$$\frac{1}{2}\left(\frac{2x+3}{x-2}\right)^{-1/2}\left(\frac{-7}{(x-2)^2}\right)$$

$$\frac{1}{2}\left(\frac{\sqrt{x-2}}{\sqrt{2x+3}}\right)\left(\frac{-7}{(x-2)^2}\right)$$

$$\frac{1}{2}\frac{(x-2)^{1/2}}{(2x+3)^{1/2}}\frac{(-7)}{(x-2)^2} = \boxed{\frac{-7}{2(2x+3)^{1/2}(x-2)^{3/2}}}$$

T.

If f(θ) = sec5θ, find f'(θ).

$$\frac{-7}{2(2x + 3)^{1/2}(x - 2)^{3/2}}$$

$$f'(\theta) = \sec 5\theta \tan 5\theta \cdot 5$$

$$= \boxed{5\sec 5\theta \tan 5\theta}$$

E.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−2	1	−1	2	4
−1	3	−2	1	1
0	−1	2	−2	−3

If $P(x) = (2f(x) + g(x))^{2/3}$, what is the value of P'(0)?

5sec5θtan5θ

$$P'(x) = \frac{2}{3}(2f(x) + g(x))^{-1/3}(2f'(x) + g'(x))$$

$$P'(0) = \frac{2}{3}(2f(0) + g(0))^{-1/3}(2f'(0) + g'(0))$$

$$= \frac{2}{3}(2(-1) + -2)^{-1/3}(2(2) + -3)$$

$$= \frac{2}{3}(-4)^{-1/3}(1) = \boxed{\frac{-2}{3\sqrt[3]{4}}}$$

N.

x	f(x)	g(x)	f'(x)	g'(x)
2	2	−1	0	−1
3	−5	4	−4	6
4	1	7	8	−2

If $q(x) = \sqrt{f(x) + g(x)}$, what is the value of q'(4)?

$$q(x) = (f(x) + g(x))^{1/2}$$

$$\frac{-2}{3\sqrt[3]{4}}$$

$$q'(x) = \frac{1}{2}(f(x) + g(x))^{-1/2}(f'(x) + g'(x))$$

$$q'(4) = \frac{1}{2}(f(4) + g(4))^{-1/2}(f'(4) + g'(4))$$

$$= \frac{1}{2}(1 + 7)^{-1/2}(8 + -2)$$

$$= \frac{1}{2}(8)^{-1/2}(6)$$

$$= 3\left(\frac{1}{\sqrt{8}}\right) = \boxed{\frac{3}{\sqrt{8}}}$$