AP Calculus
Unit 3 – Rules of Differentiation

Day 2 Notes: Finding the Derivative of a Quotient of Two Functions

Example 1: Rewrite the function $f(x)=\dfrac{2x^3-3x^2+2}{x^2}$ as a function in polynomial form. Then, find $f'(x)$.

Quotient Rule of Differentiation

To show that this rule works, let's apply this rule to the function $f(x)=\dfrac{2x^3-3x^2+2}{x^2}$ that we rewrote and differentiated as a polynomial-form above.

Example 2: We will now use the quotient rule to derive the derivative formulas for the remaining trigonometric functions. Rewrite each function in terms of sine and/or cosine and differentiate using the Quotient Rule.

$f(\theta)=\tan\theta$	$f(\theta)=\cot\theta$

$f(\theta) = \sec\theta$	$f(\theta) = \csc\theta$

Example 3: Find the derivative of each of the functions below by applying the quotient rule.

$f(x) = \dfrac{x^2 - 2x}{x+2}$	$g(x) = \dfrac{\tan x}{x+2}$
$h(\theta) = \dfrac{\sin\theta}{1-\cos\theta}$	$f(x) = \dfrac{3 - \frac{1}{x}}{x+5}$

Show, using the quotient rule, that if $f(x) = \dfrac{x^2+3x+2}{x^2-1}$, then $f'(x) = -\dfrac{3}{(x-1)^2}$.

Similar to the Product Rule, there is a very valuable lesson that we must learn when we are introduced to the quotient rule. In the box below, first factor and simplify the function, $f(x) = \dfrac{x^2+3x+2}{x^2-1}$, from above. Then, differentiate using the quotient rule

What is the lesson to be learned from the algebraic analysis above?

Let $f(x)$ and $g(x)$ be differentiable functions such that the following values are true.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	2	–1	9	–1
3	–5	–3	–4	6
4	1	7	8	–2

Estimate the value of $g'(2.5)$.

If $p(x) = \dfrac{g(x)}{f(x)}$, what is the value of $p'(4)$? What does this value say about the graph of $p(x)$ when $x = 4$? Give a reason for your answer.

If $q(x) = 2x^2\left(\dfrac{f(x)}{g(x)}\right)$, what is the value of $q'(2)$?

Find the equation of the line tangent to the graph of $v(x) = \dfrac{3x}{g(x)}$ when $x = 3$.

AP Calculus AB **Name: ________________________________**
Unit 3 – Day 2 – Assignment

For exercises 1 and 2, show the algebraic analysis that leads to the derivative of the function. Find the derivative by the specified method.

1. $f(x)=\dfrac{2x^3-3x^2+3}{x^2}$ Rewrite $f(x)$ in a polynomial-form first. Then apply the power rule to find $f'(x)$.	
2. $f(x)=\dfrac{2x^3-3x^2+3}{x^2}$ Apply the quotient rule to find $f'(x)$.	

3. Find the equation of the line tangent to the graph of $g(x)=\dfrac{2x^2-3x}{3x+1}$ when $x = -1$.

Find the derivative of each of the following functions.

4. $h(x) = \dfrac{x}{x^2+1}$	5. $h(x) = \dfrac{x}{\sqrt{x}+1}$
6. $g(\theta) = \dfrac{\cos\theta}{\theta^3}$	7. $f(\theta) = \dfrac{3(1-\sin\theta)}{2\cos\theta}$

Use the table below to complete exercises 8 – 10.

x	**$f(x)$**	**$f'(x)$**	**$g(x)$**	**$g'(x)$**
–2	**1**	**–1**	**2**	**4**
–1	**3**	**–2**	**1**	**1**
0	**–1**	**2**	**–2**	**–3**

8. If $H(x)=\dfrac{2f(x)}{g(x)}$, what is the equation of the tangent line when $x = -1$?

9. If $J(x)=\dfrac{3x+\cos x}{f(x)}$, what is the value of $J'(0)$?

10. If $K(x)=\dfrac{4x+f(x)}{3-g(x)}$, what is the slope of the normal line when $x = -2$?