AP Calculus
Unit 3 – Rules of Differentiation

Day 1 Notes: Finding the Derivative of a Product of Two Functions

Example 1: Rewrite the function $f(x) = (2x - 3)(x^2 - 2x + 1)$ as a cubic function. Then, find $f'(x)$. What does this equation of $f'(x)$ represent, again?

Product Rule of Differentiation

To show that this rule works, let's apply this rule to the function $f(x) = (2x - 3)(x^2 - 2x + 1)$ that we rewrote and differentiated as a polynomial above.

Students often wonder why this rule is so important if we could just rewrite as a polynomial and easily differentiate it. The answer to that question is simple. If it is possible to rewrite as a polynomial, always do so. But in the case of the function $g(x) = x^2 \sin x$ *, there is no way to rewrite as a polynomial.*

Example 2: Apply the product rule to find the slope of the normal line to the graph of $g(x) = x^2 \sin x$ when $x = \pi$.

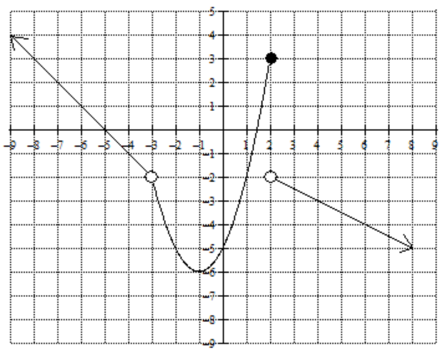
Example 3: Use the product rule to find the derivative of each of the following functions.

$f(x) = (2x^2 + 3x)(x^2 - 3)$	$g(x) = \sqrt{x}(x^2 - 3x + 2)$
$f(x) = x^3 \sin x$	$h(x) = (3x + 2)\cos x$
$g(x) = 3\theta + \theta \sin \theta$	$h(x) = \sin x \cos x$

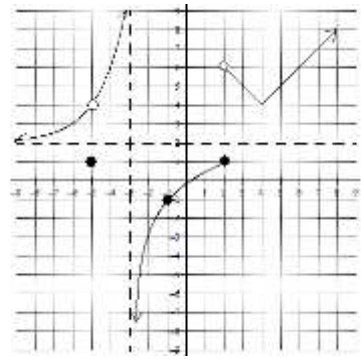
Example 4: Find the equation of the line tangent to the graph of $g(t) = t^2 \cos t$ when $t = \frac{\pi}{6}$.

Example 5: Below are graphs of two functions—$f(x)$ and $g(x)$. Let $P(x) = f(x) \cdot g(x)$ and let $R(x) = x^2 \cdot g(x)$. Use the graphs to answer the questions that follow.

Graph of $f(x)$



Graph of $g(x)$



If $g'(-4) = 2$, what is the value of $P'(-4)$?	If $R'(-2) = 20$, what is the value of $g'(-2)$?
Find the equation of the line tangent to the graph of $P(x)$ when $x = -4$.	Find the equation of the line tangent to the graph of $R(x)$ when $x = -2$.

Example 6: Let $f(x)$ and $g(x)$ be differentiable functions such that the following values are true.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
4	1	7	2	–3
3	–2	–3	–4	2
–1	2	–2	1	–1

Estimate the value of $f'(3.5)$.	If $q(x) = 2f(x) - 4g(x)$, what is the value of $q'(4)$?
If $p(x) = -2f(x)g(x)$, what is the value of $p'(3)$?	Find the equation of the line tangent to the graph of $v(x) = x^3 \cdot f(x)$ when $x = -1$.
If $k(x) = (2f(x)+3)(3-g(x))$, what is the value of $k'(3)$?	

AP Calculus AB **Name: ______________________________**
Unit 3 – Day 1 – Assignment

In the table below, a function is given. Show the algebraic analysis that leads to the derivative of the function. Find the derivative by the specified method.

1. $f(x) = (x^2 + 2x)(x - 3)$ Rewrite $f(x)$ as a polynomial first. Then apply the power rule to find $f'(x)$.	
2. $f(x) = (x^2 + 2x)(x - 3)$ Apply the product rule to find $f'(x)$.	

For exercises 3 – 5, find the derivative of each function.

3. $f(x) = (x^2 + 2)(x^2 - 2x)$	5. $f(x) = \sqrt[3]{x}(x^2 + 4)$
4. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$	

Find the slope of the normal line drawn to the graph of each function at the indicated value of *x*.

6. $g(x) = \sqrt{x}\sin x$ when $x = \pi$	7. $h(x) = \sin x(\sin x + \cos x)$ when $x = \frac{\pi}{4}$

For each of the functions below, find the equation of the tangent line drawn to the graph of $g(x)$ at the indicated value of *x*.

8. $g(x) = \sqrt{x}\left(2x^2 - 4\right)$ when $x = 4$	9. $g(x) = x^2 \cos x$ when $x = \frac{\pi}{2}$

Use the table below to complete exercises 10 – 12.

x	**$f(x)$**	**$f'(x)$**	**$g(x)$**	**$g'(x)$**
–2	**1**	**–1**	**2**	**4**
–1	**3**	**–2**	**1**	**1**
0	**–1**	**2**	**–2**	**–3**

10. If $H(x) = 2f(x) \cdot g(x)$, what is the equation of the tangent line when $x = -1$?

11. If $J(x) = g(x) \cdot \sin x$, what is the value of $J'(0)$?

12. If $K(x) = (4x - f(x))(2g(x) - 2)$, what is the slope of the normal line when $x = -2$?