AP Calculus AB Name: Answer Key*
Unit 1 – Limits & Continuity - REVIEW

Multiple Choice Practice

1. $\lim_{x\to 0}\frac{4x-3}{7x+1}=$ $\frac{4(0)-3}{7(0)+1}=\frac{-3}{1}=-3$

A. ∞ B. $-\infty$ C. 0 D. $\frac{4}{7}$ E. -3

2. $\lim_{x\to\frac{1}{3}}\frac{9x^2-1}{3x-1}=$ $\cancel{3(\frac{1}{3})-1}$ $\frac{(3x+1)\cancel{(3x-1)}}{\cancel{(3x-1)}}$ $\lim_{x\to 1/3}(3x+1)=3(\frac{1}{3})+1=1+1=2$

A. ∞ B. $-\infty$ C. 0 D. 2 E. 3

3. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}=$ $\cancel{(2)^2-4}$ $\frac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}\to\lim_{x\to 2}\frac{(x^2+2x+4)}{x+2}=\frac{(2)^2+2(2)+4}{2+2}=\frac{12}{4}=3$

A. 4 B. 0 C. 1 D. 3 E. 2

4. The function $G(x)=\begin{cases}x-3, & x>2\\ -5, & x=2\\ 3x-7, & x<2\end{cases}$ is not continuous at $x=2$ because…

$G(2)=-5$, $G(2)$ is defined

$\lim_{x\to 2}G(x)=-1$ $\lim_{x\to 2^+}G(x)=2-3=-1$ $\lim_{x\to 2^-}G(x)=3(2)-7=-1$

~~A.~~ $G(2)$ is not defined ~~B.~~ $\lim_{x\to 2}G(x)$ does not exist C. $\lim_{x\to 2}G(x)\neq G(2)$

D. Only reasons B and C E. All of the above reasons.

$-1\neq -$

5. $\lim_{x\to\infty}\frac{-3x^2+7x^3+2}{2x^3-3x^2+5}=$ $\frac{-\frac{3x^2}{x^3}+\frac{7x^3}{x^3}+\frac{2}{x^3}}{\frac{2x^3}{x^3}-\frac{3x^2}{x^3}+\frac{5}{x^3}}$ $\lim_{x\to\infty}\frac{-\frac{3}{x}+7+\frac{2}{x^3}}{2-\frac{3}{x}+\frac{5}{x^3}}=\frac{0+7+0}{2-0+0}=\frac{7}{2}$

A. ∞ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$

6. $\lim_{x\to -2}\frac{\sqrt{2x+5}-1}{x+2}=$ $\frac{(\sqrt{2x+5}-1)}{x+2}\circ\frac{(\sqrt{2x+5}+1)}{\sqrt{2x+5}+1}=\frac{2x+5-1}{(x+2)(\sqrt{2x+5}+1)}=\frac{2x+4}{(x+2)(\sqrt{2x+5}+1)}=\frac{2\cancel{(x+2)}}{\cancel{(x+2)}(\sqrt{2x+5}+1)}$

A. 1 B. 0 C. ∞ D. $-\infty$ E. Does Not Exist

$\lim_{x\to -2}\frac{2}{\sqrt{2x+5}+1}=\frac{2}{\sqrt{2(-2)+5}+1}=\frac{2}{1+1}=\frac{2}{2}=1$

7. If $f(x) = 3x^2 - 5x$, then find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

A. $3x - 5$
B. $6x - 5$
C. $6x$
D. 0
E. Does not exist

$f(x+h) = 3(x+h)^2 - 5(x+h)$
$= 3(x^2 + 2xh + h^2) - 5x - 5h$
$= 3x^2 + 6xh + 3h^2 - 5x - 5h - (3x^2 - 5x)$
$= \cancel{3x^2} + 6xh + 3h^2 - \cancel{5x} - 5h - \cancel{3x^2} + \cancel{5x}$
$= \frac{6xh + 3h^2 - 5h}{h} = \frac{\cancel{h}(6x + 3h - 5)}{\cancel{h}}$

$\lim_{h \to 0} (6x + 3h - 5) = 6x + 3(0) - 5 = 6x - 5$

$\sqrt{x^2} = -x$
$x < 0$

8. $\lim_{x \to -\infty} \frac{2 - 5x}{\sqrt{x^2 + 2}} = \frac{\frac{2}{-x} - \frac{5x}{-x}}{\sqrt{\frac{x^2}{x^2} + \frac{2}{x^2}}}$ $\lim_{x \to -\infty} \frac{-\frac{2}{x} + 5}{\sqrt{1 + \frac{2}{x^2}}} = \frac{0 + 5}{\sqrt{1 + 0}} = \frac{5}{\sqrt{1}} = 5$

A. 5 B. -5 C. 0 D. $-\infty$ E. ∞

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9. The function $f(x) = \frac{2x^2 + x - 3}{x^2 + 4x - 5}$ has a vertical asymptote at $x = -5$ because…

-6 3 -2

$(2x^2 + 3x) - 2x - 3$
$x(2x+3) - 1(2x+3)$

$\lim_{x \to -5^+} \to \frac{2(-5.1) + 3}{-5.1 + 5} = \frac{\ominus}{\ominus} = +\infty$

~~A.~~ $\lim_{x \to -5^+} f(x) = \infty$ ~~B.~~ $\lim_{x \to -5^-} f(x) = -\infty$

$\frac{(2x+3)\cancel{(x-1)}}{(x+5)\cancel{(x-1)}}$

C. $\lim_{x \to -5^-} f(x) = \infty$ ~~D.~~ $\lim_{x \to \infty} f(x) = -5$

$\lim_{x \to -5^+} \frac{(2x+3)}{x+5} = \frac{2(-4.9) + 3}{-4.9 + 5} = \frac{\ominus}{\oplus} = -\infty$

E. $f(x)$ does not have a vertical asymptote at $x = -5$

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10. Consider the function $H(x) = \begin{cases} 3x - 5, & x < 3 \\ x^2 - 2x, & x \geq 3 \end{cases}$. Which of the following statements is/are true?

✓ I. $\lim_{x \to 3^-} H(x) = 4$.

~~II.~~ $\lim_{x \to 3} H(x)$ exists. False

~~III.~~ $H(x)$ is continuous at $x = 3$. False, since $\lim_{x \to 3}$ DNE

A. I only B. II only C. I and II only

D. I, II and III E. None of these statements is true

$\lim_{x \to 3^-} H(x) = 3(3) - 5 = 9 - 5 = 4$

$\lim_{x \to 3^+} H(x) = (3)^2 - 2(3) = 9 - 6 = 3$

Free Response Practice #1
Calculator Permitted

Consider the function $h(x) = \dfrac{-2x - \sin x}{x - 1}$ to answer the following questions.

a. Find $\lim_{x \to 1^+} h(x)$. Show your numerical analysis that leads to your answer and explain what this result implies graphically about $h(x)$ at $x = 1$.

$\lim_{x \to 1^+} h(x)$

x	$h(x)$
1.1	$\dfrac{-2(1.1)-\sin(1.1)}{1.1-1} = \dfrac{\ominus}{\oplus} = -\infty$

$$\boxed{\lim_{x \to 1^+} h(x) = -\infty}$$

Since $\lim_{x \to 1^+} h(x) = -\infty$, then $x=1$ is a vertical asymptote of $h(x)$.

b. Find $\lim_{x \to \frac{\pi}{2}} [h(x) \cdot (2x - 2)]$. Show your analysis.

$$\lim_{x \to \pi/2} h(x) \cdot \lim_{x \to \pi/2} (2x-2)$$

$$\frac{-2(\frac{\pi}{2})-\sin(\frac{\pi}{2})}{\frac{\pi}{2}-1} \cdot 2\left(\frac{\pi}{2}\right)-2$$

$$\frac{-\pi-1}{\frac{\pi}{2}-1} \cdot (\pi-2)$$

$$= \frac{-\pi-1}{\frac{\pi-2}{2}} \cdot \pi-2$$

$$= -\pi-1 \cdot \frac{2}{\cancel{\pi-2}} \cdot \cancel{\pi-2}$$

$$= (-\pi-1)(2) = \boxed{-2\pi-2}$$

OR $\boxed{-8.283}$

(0,1)

c. Explain why the Intermediate Value Theorem guarantees a value of c on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find c.

① $h(x)$ is only discont. at $x=1$ (vert. asymptote)
So, $h(x)$ is continuous on $[1.5, 2.5]$

② $f(1.5) = \dfrac{-2(1.5)-\sin(1.5)}{1.5-1} = -7.995$

$f(2.5) = \dfrac{-2(2.5)-\sin(2.5)}{2.5-1} = -3.732$

$h(c) = -4$ is between $h(1.5)$ and $h(2.5)$,

$\therefore$ The IVT is applicable for $h(x)$ on $[1.5, 2.5]$.

$$\underbrace{\frac{-2c-\sin c}{c-1}}_{y_1} = \underbrace{-4}_{y_2}$$

2nd Trace Intersect $\boxed{c = 2.354}$