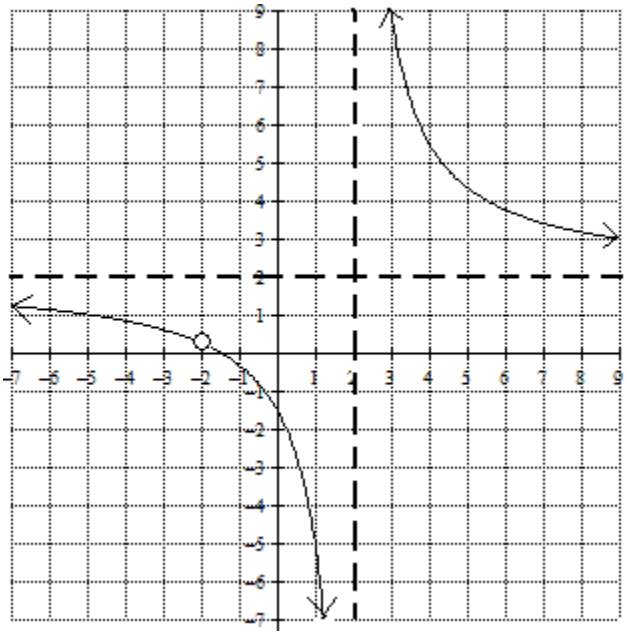
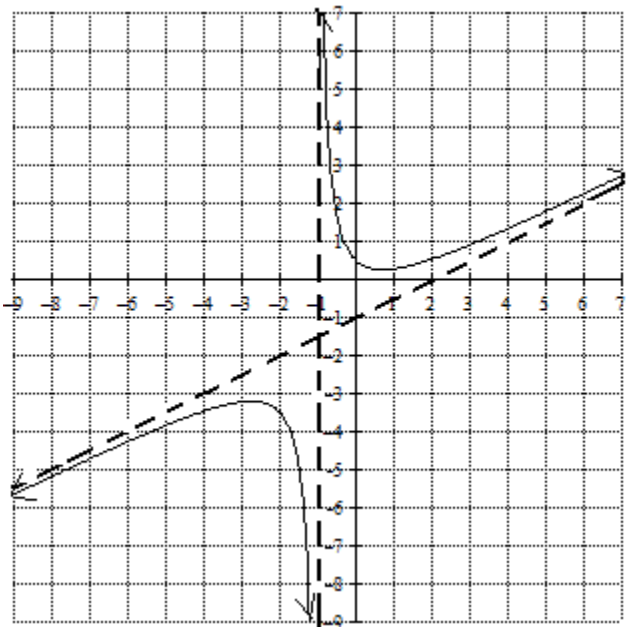
AP Calculus
Unit 1 – Limits & Continuity

Day 6 Notes: Infinite Limits & Limits at Infinity

$$f(x)=\frac{2x^2+7x+6}{x^2-4}$$



$$g(x)=\frac{x^2-x+1}{2x+2}$$



Infinite Limits	**Limits at Infinity**
$\lim_{x\to 2^-} f(x) =$	$\lim_{x\to\infty} f(x) =$
$\lim_{x\to -1^-} g(x) =$	$\lim_{x\to\infty} g(x) =$

****<u>Justification of the Existence of a Vertical Asymptote Using Limits</u>:**

<u>Example 1</u>: Find any vertical asymptote(s) that exist. Justify your answer(s) using a limit(s).

$$h(x) = \frac{2x^2 + 7x + 3}{x^2 + 2x - 3}$$

<u>Example 2</u>: Find the limit indicated. Explain what the result of the limit means about the graph of the given rational function.

$$\lim_{x \to -1^+} \frac{x^2 - x + 1}{2x + 2}$$

<u>Example 3</u>: Find each of the following limits at infinity. Give an explanation of your reasoning for each.

$\lim_{x \to \infty} -3x^3 + x - 4$	$\lim_{x \to -\infty} (4 - x)^2 (x - 3)(x + 1)$	$\lim_{x \to \infty} \frac{3 - 2x}{x + 3}$

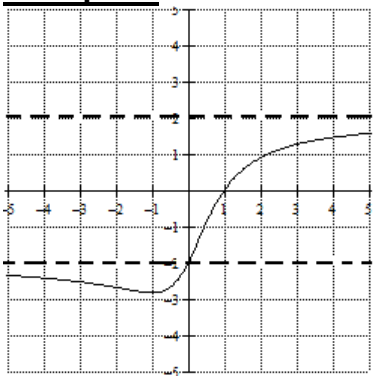
Algebraic Analysis to Evaluate a Limit at Infinity for Rational Functions:

Divide every term in both the numerator and the denominator by the highest power of *x* that appears in the denominator. Then, evaluate the indicated limit.

Example 4: Find each of the following limits at infinity. Show your limit analysis. Then, explain why the result of the limit concurs with your graphical understanding of asymptotic behavior of the rational function.

$\lim_{x\to\infty} \frac{x+2}{x^2+5x+6}$	$\lim_{x\to\infty} \frac{2x^2+7x+6}{x^2+5x+6}$
$\lim_{x\to-\infty} \frac{2x^2+7x+6}{x^2+5x+6}$	$\lim_{x\to-\infty} \frac{x^2+3x+2}{x-1}$

Example 5:



Use the graph to find each of the following limits.

$\lim_{x\to-\infty} \frac{2x-2}{\sqrt{x^2+1}} =$ _____ $\lim_{x\to\infty} \frac{2x-2}{\sqrt{x^2+1}} =$ _____

Perform the same algebraic analysis that we did earlier to find the limits at infinity. The only problem that we will encounter is what to do when $x \to -\infty$.

$\lim_{x\to\infty} \frac{2x-2}{\sqrt{x^2+1}}$	$\lim_{x\to-\infty} \frac{2x-2}{\sqrt{x^2+1}}$

Example 6: Find each of the following limits at infinity. What do the results show about the existence of a horizontal asymptote? Justify your reasoning.

$\lim_{x\to-\infty} \frac{3x-2}{\sqrt{2x^2+1}}$	$\lim_{x\to\infty} \frac{x^2+2x}{\sqrt{3x^2+2}}$

AP Calculus AB **Name: _____**
Unit 1 – Day 6 – Assignment

For exercises 1 – 3, find the limit indicated. Explain what the result of the limit means about the graph of the given rational function.

1. $\lim_{x\to -5^+} \dfrac{x^2-x-6}{x+5}$	2. $\lim_{x\to -2^-} \dfrac{x-5}{x^2+x-2}$	3. $\lim_{x\to 2^-} \dfrac{2x^2+x-3}{x^2-3x+2}$

Find each of the following limits at infinity. Show your limit analysis. Then, explain why the result of the limit concurs with your graphical understanding of asymptotic behavior of the rational function.

4. $\lim_{x\to -\infty} \dfrac{3x+2-5x^2}{2x^2-3x-1}$	5. $\lim_{x\to \infty} \dfrac{3x+5}{2x^2-3x}$	6. $\lim_{x\to -\infty} \dfrac{-2x^2+5}{3x+2}$

Find each of the following limits at infinity. Explain how you arrived at your answer.

7. $\lim_{x\to-\infty} -3x^3 - 2x + 4$	8. $\lim_{x\to\infty} (2-x)(x+2)^3$
9. $\lim_{x\to\infty} \left(\frac{3}{2}\right)^{-x+2} - 2$	10. $\lim_{x\to-\infty} -\left(\frac{2}{3}\right)^{-x+1} + 3$

Find each of the following limits at infinity. What do the results show about the existence of a horizontal asymptote? Justify your reasoning.

11. $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2 - x}}$	12. $\lim_{x\to\infty} \frac{-2x^2 + x}{\sqrt{2x^2 - 3}}$