AP Calculus AB Name: Answer Key*

Unit 1 – Day 6 – Assignment

For exercises 1 – 3, find the limit indicated. Explain what the result of the limit means about the graph of the given rational function.

1. $\lim_{x\to -5^{+}} \frac{x^2-x-6}{x+5}$ $\frac{(x-3)(x-2)}{(x+5)}$ Plug in $x=-4.9$ $\frac{(-4.9-3)(-4.9-2)}{(-4.9+5)} = \frac{\ominus\ominus}{\oplus} = \frac{\oplus}{\oplus} = \boxed{\infty}$ Since $\lim_{x\to -5^+} f(x)=\infty$, then $x=-5$ is a vertical asymptote	2. $\lim_{x\to -2^{-}} \frac{x-5}{x^2+x-2}$ $\frac{(x-5)}{(x+2)(x-1)}$ plug in $x=-2.1$ $\frac{(-2.1-5)}{(-2.1+2)(-2.1-1)} = \frac{\ominus}{\ominus(\ominus)} = \frac{\ominus}{\oplus} = \boxed{-\infty}$ Since $\lim_{x\to -2^-} f(x)$ is $-\infty$, then $x=-2$ is a vert asymp.	3. $\lim_{x\to 2^{-}} \frac{2x^2+x-3}{x^2-3x+2}$ $\frac{\cancel{(x-1)}(2x+3)}{(x-2)\cancel{(x-1)}}$ plug in $x=1.9$ $\frac{2(1.9)+3}{1.9-2} = \frac{\oplus}{\ominus} = \boxed{-\infty}$ Since $\lim_{x\to 2^-} f(x) = -\infty$, then $x=2$ is a vert. asymp

$3 \cdot -2 = -6$
$2x^2+3x-2x-3$
$x(2x+3)+-1(2x+3)$

Find each of the following limits at infinity. Show your limit analysis. Then, explain why the result of the limit concurs with your graphical understanding of asymptotic behavior of the rational function.

4. $\lim_{x\to -\infty} \frac{3x+2-5x^2}{2x^2-3x-1}$ $\frac{\frac{3x}{x^2}+\frac{2}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}-\frac{1}{x^2}}$ $\lim_{x\to-\infty} \frac{\frac{3}{x}+\frac{2}{x^2}-5}{2-\frac{3}{x}-\frac{1}{x^2}}$ $= \frac{0+0-5}{2-0-0} = \boxed{\frac{-5}{2}}$ Since the degrees of numer. & den. are equal, then the graph has a horiz. asymptote at $y=-5/2$	5. $\lim_{x\to\infty} \frac{3x+5}{2x^2-3x}$ $\frac{\frac{3x}{x^2}+\frac{5}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}}$ $\lim_{x\to\infty} \frac{\frac{3}{x}+\frac{5}{x^2}}{2-\frac{3}{x}} = \frac{0+0}{2-0} = \frac{0}{2} = \boxed{0}$ Since the degree of the num < deg of den, then $y=0$ is a horizontal asymp	6. $\lim_{x\to -\infty} \frac{-2x^2+5}{3x+2}$ $\frac{\frac{-2x^2}{x}+\frac{5}{x}}{\frac{3x}{x}+\frac{2}{x}}$ $\lim_{x\to-\infty} \frac{-2x+\frac{5}{x}}{3+\frac{2}{x}} = \frac{-2(-\infty)+0}{3+0} = \frac{\infty}{3} = \boxed{\infty}$ Since the degree of num > deg of denom, there is no horiz. asympt but there is a slant asympt w/ a neg. slope.

Find each of the following limits at infinity. Explain how you arrived at your answer.

7. $\lim_{x\to-\infty}(-3x^3-2x+4)$

$\boxed{\infty}$

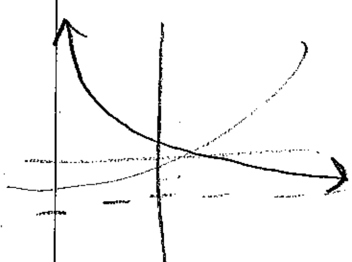
Degree is odd & leading coeff is negative so graph will look like

8. $\lim_{x\to\infty}(2-x)(x+2)^3$ $\quad -x^4$ $\quad \boxed{-\infty}$

Degree is even & leading coeff is negative, so graph will look like

9. $\lim_{x\to\infty}\left(\frac{3}{2}\right)^{-x+2}-2$

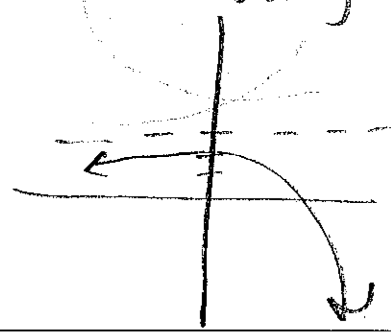
growth



$\boxed{-2}$

10. $\lim_{x\to-\infty}-\left(\frac{2}{3}\right)^{-x+1}+3$

decay



$\boxed{3}$

Find each of the following limits at infinity. What do the results show about the existence of a horizontal asymptote? Justify your reasoning.

11. $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-x}}$ $\quad \sqrt{x^2}=-x$

$$\frac{\frac{2x}{-x}+\frac{1}{-x}}{\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}}$$

$$\lim_{x\to-\infty}\frac{-2+\frac{1}{-x}}{\sqrt{1-\frac{1}{x}}}$$

$$=\frac{-2+0}{\sqrt{1-0}}=\frac{-2}{1}=\boxed{-2}$$

Since $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-x}}=-2$, the graph has a horizontal asymptote at $y=-2$.

12. $\lim_{x\to\infty}\frac{-2x^2+x}{\sqrt{2x^2-3}}$

$$\frac{\frac{-2x^2}{x}+\frac{x}{x}}{\sqrt{\frac{2x^2}{x^2}-\frac{3}{x^2}}}$$

$$\lim_{x\to\infty}\frac{-2x+1}{\sqrt{2-\frac{3}{x^2}}}$$

$$\frac{-2(\infty)+1}{\sqrt{2-0}}=\boxed{-\infty}$$

Since $\lim_{x\to\infty}\frac{-2x^2+x}{\sqrt{2x^2-3}}\neq$ a real #, the graph does not have a horiz. asymptote.