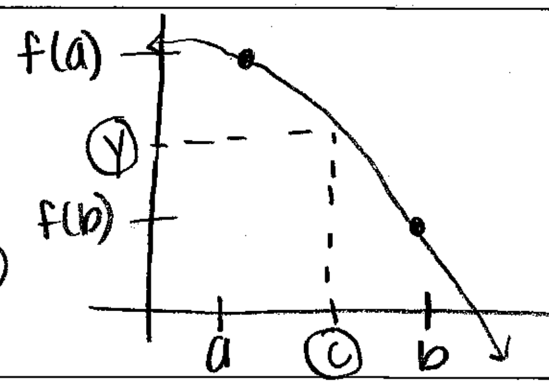
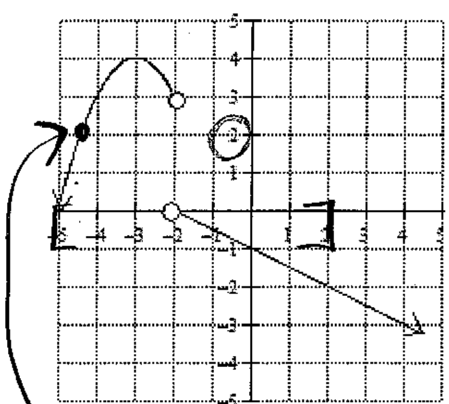
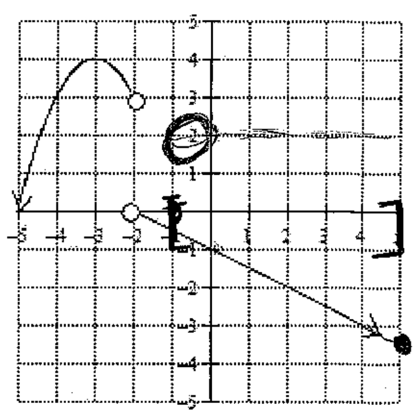
Day 5 Notes: Intermediate Value Theorem

Intermediate Value Theorem

If f(x) is continuous on [a,b] and f(a) < y < f(b) or f(a) > y > f(b), then there exists at least one value, x = c, on (a,b) such that f(c) = y.



Example 1: Investigate the graphs below to determine if the theorem is applicable for these functions on the specified intervals for the values given.

$f(x)=\begin{cases}-(x+3)^2+4, & x<-2\\ -\frac{1}{2}x-1, & x>-2\end{cases}$ 	$f(x)=\begin{cases}-(x+3)^2+4, & x<-2\\ -\frac{1}{2}x-1, & x>-2\end{cases}$ 
Is there a value of c on $[-5, 2]$ such that $f(c) = 2$? Yes, there is a y-value of 2 b/t [-5,2]	Is there a value of c $[-1, 5]$ such that $f(c) = 2$? NO, no y-value of 2 b/t [-1,5]
Does the I.V.T. guarantee a value of c such that $f(c) = 2$ on the interval $[-5, 2]$? Why or why not? NO, the IVT does NOT guarantee a value of c on [-5,2] b/c f(x) is NOT continuous on this interval.	Does the I.V.T. guarantee a value of c such that $f(c) = 2$ on the interval $[-1, 5]$? Why or why not? No, the IVT does not guarantee such a value of c on [-1,5] b/c f(c)=2 is not b/t f(-1)=0 and f(5)=-3.5.

What **two conditions** must be true to verify the applicability of the Intermediate Value Theorem?

1. f(x) must be continuous on [a,b]
2. f(c) must be between f(a) and f(b)

Example 2: For each of the following functions, determine if the I.V.T. is applicable or not and state why or why not. Then, if it is applicable, find the value of *c* guaranteed to exist by the theorem.

a. $f(x)=\frac{x-3}{x+2}$ on the interval [–1, 3] for $f(c)=\frac{2}{3}$

① f(x) is only discontinuous at $x=-2$, which is not on $[-1,3]$. Thus, f(x) is continuous on $[-1,3]$. ✓

② $f(-1)=\frac{-1-3}{-1+2}=\frac{-4}{1}=-4$

$f(3)=\frac{3-3}{3+2}=\frac{0}{5}=0$

$f(c)=2/3$ is NOT between $f(-1)=-4$ & $f(3)=0$.

∴ The IVT is not applicable for $f(c)=\frac{2}{3}$ on $[-1,3]$

b. $f(x)=\frac{x-3}{x+2}$ on the interval [–4, 1] for $f(c)=\frac{2}{3}$

① f(x) is discontinuous at $x=-2$. Thus, f(x) is not continuous on $[-4,1]$.

∴ The IVT is not applicable for $f(c)=\frac{2}{3}$ on $[-4,1]$.

c. $f(x)=\frac{x}{x-2}$ on the interval [–1, 1] for $f(c)=-\frac{1}{2}$

① f(x) is discont. at $x=2$, ∴ f(x) is continuous on $[-1,1]$ ✓

② $f(-1)=\frac{1}{-1-2}=\frac{1}{-3}$

$f(1)=\frac{1}{1-2}=\frac{1}{-1}=-1$

$f(c)=-1/2$ is b/t $f(-1)$ & $f(1)$ ✓

∴ The IVT is applicable for $f(c)=-1/2$ on $[-1,1]$.

$\frac{c}{c-2}=\frac{-1}{2}$

$2c=-c+2$

$3c=2$

$\boxed{c=2/3}$

d. $f(x)=-\left(\frac{1}{2}\right)^{-x+3}-2$ on the interval [3, 5] for $f(c)=-4$

① f(x) is cont on $(-\infty,\infty)$. f(x) is cont on $[3,5]$ ✓

② $f(3)=-\left(\frac{1}{2}\right)^{-3+3}-2=-\left(\frac{1}{2}\right)^{0}-2=-1-2=-3$

$f(5)=-\left(\frac{1}{2}\right)^{-5+3}-2=-\left(\frac{1}{2}\right)^{-2}-2=-(2)^2-2=-4-2=-6$

$f(c)=-4$ is b/t $f(3)$ & $f(5)$ ✓

∴ The IVT is applicable.

$-\left(\frac{1}{2}\right)^{-c+3}-2=-4$

$\left(\frac{1}{2}\right)^{-c+3}=2$

$\frac{1}{2}^{x}=2$

$x=-1$

$\log_{\frac{1}{2}}(2)=-c+3$

$-1=-c+3$

$-4=-c$ $\boxed{c=4}$