AP Calculus AB
Unit 1 – Day 5 – Assignment

Name: Answer Key*

$f(c) = 0$

1. Determine, using the intermediate value theorem, if the function $F(x) = x^3 + 2x - 1$ has a zero on the interval [0, 1]. Justify your answer and find the indicated zero, if it exists.

① f(x) is continuous on [0, 1] b/c all cubic functions are cont.

② $F(0) = (0)^3 + 2(0) - 1 = -1$
$F(1) = (1)^3 + 2(1) - 1 = 2$ } 0 is b/t −1 & 2

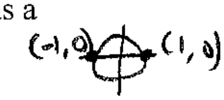
Since $F(0) < F(c) = 0 < F(1)$, then the IVT guarantees a value of c.

$c^3 + 2c - 1 = 0$ ← use calc. to find the zero

$\boxed{c = .453}$

2. Determine, using the intermediate value theorem, if the function $g(\theta) = \theta^2 - 2 - \cos\theta$ has a zero on the interval $[0, \pi]$. Justify your answer and find the indicated zero, if it exists.

$f(c) = 0$



① $g(\theta)$ is continuous b/c $g(\theta)$ is never undefined.

② $g(0) = 0^2 - 2 - \cos(0) = 0 - 2 - 1 = -3$
$g(\pi) = \pi^2 - 2 - \cos(\pi) = \pi^2 - 2 - (-1) = \pi^2 - 1 \approx 8.86$ } 0 is b/t −3 & 8.86
g(c) is b/t g(0) & g(π)

∴ The IVT is applicable for f(c) = 0 on [0, π].

$c^2 - 2 - \cos c = 0$ ← use graph on calc

not b/t [0, π] → $\cancel{c = -1.455}$ $\boxed{c = 1.455}$

For exercises 3 – 5, first, verify that the I.V.T. is applicable for the given function on the given interval. Then, if it is applicable, find the value of the indicated *c*, guaranteed by the theorem.

3. $f(x) = x^2 - 6x + 8$ Interval: [0, 3] $f(c) = 0$

① f(x) is a quadratic function that is always continuous ✓

② $f(0) = (0)^2 - 6(0) + 8 = 8$
$f(3) = (3)^2 - 6(3) + 8 = -1$ } 0 is b/t −1 & 8
f(c) is b/t f(0) & f(3) ✓

∴ The IVT is applicable for f(c) = 0 on [0, 3].

$c^2 - 6c + 8 = 0$
$(c - 4)(c - 2) = 0$
$\cancel{c = 4}$ $\boxed{c = 2}$
↑
not b/t [0, 3]

4. $g(x) = x^3 - x^2 + x - 2$ Interval: $[0, 3]$ $g(c) = 4$

① $g(x)$ is a cubic function that is always continuous. ✓

② $g(0) = (0)^3 - (0)^2 + 0 - 2 = -2$

$g(3) = (3)^3 - (3)^2 + 3 - 2 = 19$

4 is b/t -2 & 19

$g(c)$ is b/t $g(0)$ & $g(3)$

∴ The IVT is applicable for $g(c) = 4$ on $[0, 3]$.

$c^3 - c^2 + c - 2 = 4$

$c^3 - c^2 + c - 6 = 0$ ← use graph on calc to find zero.

$\boxed{c = 2}$

5. $h(x) = \dfrac{x^2 + x}{x - 1}$ Interval: $\left[\frac{5}{2}, 4\right]$ $h(c) = 6$

① $h(x)$ is discont. only at $x = 1$. $h(x)$ is cont on $[\frac{5}{2}, 4]$ ✓

② $h(\frac{5}{2}) = \dfrac{(\frac{5}{2})^2 + (\frac{5}{2})}{(\frac{5}{2}) - 1} = \dfrac{8.75}{1.5} = 5.833$

$h(4) = \dfrac{(4)^2 + 4}{4 - 1} = \dfrac{16 + 4}{3} = 6.667$

6 is b/t 5.833 & 6.667

↓

$h(c)$ is b/t $h(5/2)$ & $h(4)$

∴ The IVT is applicable for $h(c) = 6$ on $[5/2, 4]$

$\dfrac{c^2 + c}{c - 1} = \dfrac{6}{1}$ → $c^2 + c = 6c - 6$

$c^2 - 5c + 6 = 0$

$(c - 3)(c - 2)$

$\boxed{c = 3}$ $\cancel{c = 2}$ not b/t $[5/2, 4]$