Day 4: Limit-Based Continuity

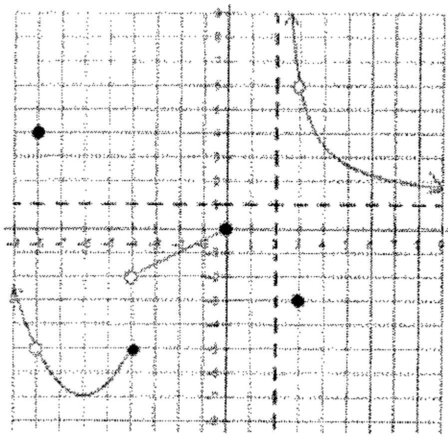
Three Part **Definition of Continuity** to determine if a function, f(x), is continuous or not at x = a.

I. $f(a)$ is defined

II. $\lim_{x \to a} f(x)$ exists

III. $f(a) = \lim_{x \to a} f(x)$

The graph of the function, $G(x)$, pictured to the right has several x – values at which the function is not continuous. For each of the following x – values, use the three part definition of continuity to determine if the function is continuous or not.



1. $x = -8$

I. $G(-8) = 4$

$G(-8)$ is defined. ✓

II. $\lim_{x \to -8^-} G(x) = -5$

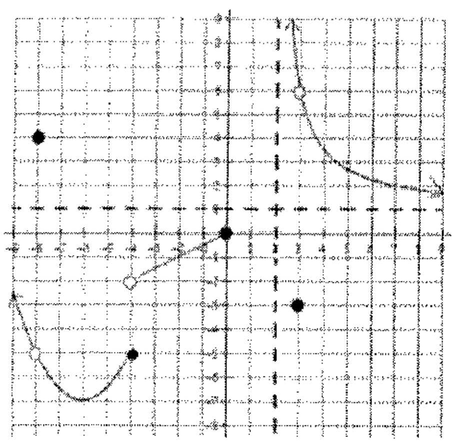
$\lim_{x \to -8^+} G(x) = -5$

Since, $\lim_{x \to -8^-} G(x) = \lim_{x \to -8^+} G(x) = -5$, $\lim_{x \to -8} G(x)$ exists. ✓

III. Since $G(-8) = 4$ and $\lim_{x \to -8} G(x) = -5$, $G(-8) \neq \lim_{x \to -8} G(x)$.

$\boxed{\therefore G(x) \text{ is } \underline{\text{NOT}} \text{ continuous at } x = -8.}$

The graph of the function, $G(x)$, pictured to the right has several x – values at which the function is not continuous. For each of the following x – values, use the three part definition of continuity to determine if the function is continuous or not.



2. $x = -6$

I. $G(-6) = -7$, $G(-6)$ is defined ✓

II. $\lim_{x \to -6^-} G(x) = -7$

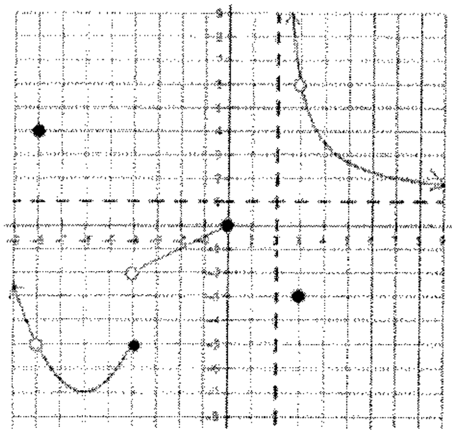
$\lim_{x \to -6^+} G(x) = -7$

Since, $\lim_{x \to -6^-} G(x) = \lim_{x \to -6^+} G(x) = -7$, $\lim_{x \to -6} G(x)$ exists. ✓

III. $G(-6) = \lim_{x \to -6} G(x) = -7$ ✓

$\boxed{\therefore G(x) \text{ is continuous at } x = -6}$

The graph of the function, $G(x)$, pictured to the right has several $x-$ values at which the function is not continuous. For each of the following $x-$ values, use the three part definition of continuity to determine if the function is continuous or not.



3. $x=-4$

I. $G(-4)=-5$, $G(-4)$ is defined. ✓

II. $\lim_{x\to-4^-} G(x) = -5$

$\lim_{x\to-4^+} G(x) = -2$

Since, $\lim_{x\to-4^-} G(x) \neq \lim_{x\to-4^+} G(x)$, then $\lim_{x\to-4} G(x)$ D.N.E.

$\boxed{\therefore\ G(x) \text{ is } \underline{\text{NOT}} \text{ continuous at } x=-4}$

Use the three part definition of continuity to determine if the given functions are continuous at the indicated values of x.

4. $f(x)=\begin{cases} -2\sqrt{x+6}, & x<-2 \\ 3x+2, & x=-2 \\ e^x+\cos(\pi x), & x>-2 \end{cases}$ at $x=-2$

I. $f(-2)=3(-2)+2=-4$, $f(-2)$ is defined ✓

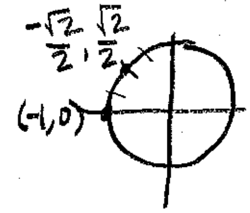
II. $\lim_{x\to-2^-} f(x) = -2\sqrt{-2+6} = -2(2) = -4$

$\lim_{x\to-2^+} f(x) = e^{-2} + \cos(-2\pi) = \frac{1}{e^2} + \cos(0) = \frac{1}{e^2}+1$

$\lim_{x\to-2} f(x)$ D.N.E b/c $\lim_{x\to-2^-} f(x) \neq \lim_{x\to-2^+} f(x)$

$\boxed{\therefore\ f(x) \text{ is } \underline{\text{NOT}} \text{ continuous at } x=-2}$

Use the three part definition of continuity to determine if the given functions are continuous at the indicated values of x.



5. $g(x) = \begin{cases} e^x \cos x, & x < \pi \\ e^x \tan\left(\frac{3x}{4}\right), & x \geq \pi \end{cases}$ at $x = \pi$

I. $g(\pi) = e^{\pi}\tan\left(\frac{3\pi}{4}\right) = e^{\pi}(-1) = -e^{\pi}$

II. $\lim_{x \to \pi^-} g(x) = e^{\pi}\cos(\pi) = e^{\pi}(-1) = -e^{\pi}$

$\lim_{x \to \pi^+} g(x) = -e^{\pi}$

$\lim_{x \to \pi} g(x) = -e^{\pi}$ since $\lim_{x \to \pi^-} g(x) = \lim_{x \to \pi^+} g(x) = -e^{\pi}$.

III. $g(\pi) = \lim_{x \to \pi} g(x) = -e^{\pi}$

$\boxed{\therefore g(x) \text{ is continuous at } x = \pi}$

6. Consider the function, $f(x)$, to the right to answer the following questions.

$$f(x) = \begin{cases} 2, & x \leq -1 \\ mx + k, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$$

a. What two limits must equal in order for $f(x)$ to be continuous at $x = -1$?

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) \longrightarrow 2 = m(-1) + k$

b. What two limits must equal in order for $f(x)$ to be continuous at $x = 3$?

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) \longrightarrow m(3) + k = -2$

c. Determine the values of m and k so that the function is continuous everywhere.

$-m + \cancel{k} = 2$

$+ \quad -3m - \cancel{k} = +2$

$-4m = 4$

$\boxed{m = -1}$

$-m + k = 2$

$-(-1) + k = 2$

$\boxed{k = 1}$