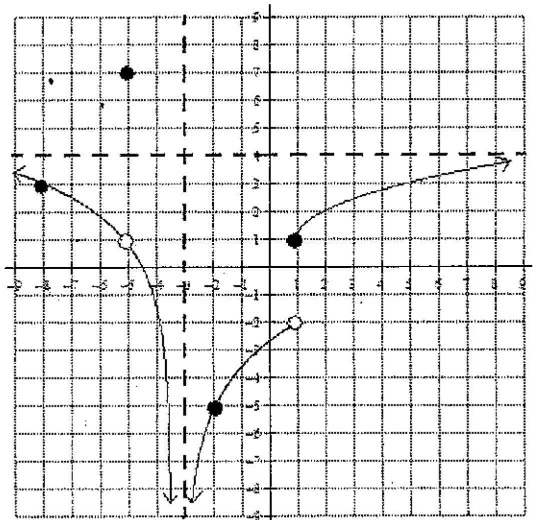
AP Calculus AB Name: Answer Key*

Unit 1 – Day 4 – Assignment

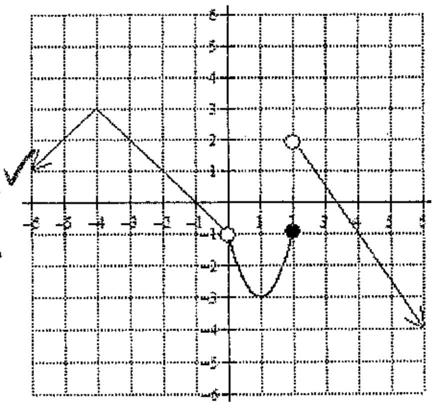
Limit – Based Continuity

For exercises 1 – 3, determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



1. $x = -5$	2. $x = 1$	3. $x = -2$
I. $f(-5) = 7$, $f(-5)$ is defined ✓ II. $\lim_{x \to -5^-} f(x) = 1$ $\lim_{x \to -5^+} f(x) = 1$ Since $\lim_{x \to -5^-} f(x) = \lim_{x \to -5^+} f(x)$, $\lim_{x \to -5} f(x)$ exists. ✓ III. $f(-5) \neq \lim_{x \to -5} f(x)$ ∴ $f(x)$ is NOT continuous at $x = -5$	I. $f(1) = 1$, $f(1)$ is defined ✓ II. $\lim_{x \to 1^-} f(x) = -2$ $\lim_{x \to 1^+} f(x) = 1$ Since $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$ $\lim_{x \to 1} f(x)$ D.N.E ∴ $f(x)$ is NOT continuous at $x = 1$.	I. $f(-2) = -5$ $f(-2)$ is defined ✓ II. $\lim_{x \to -2^-} f(x) = -5$ $\lim_{x \to -2^+} f(x) = -5$. $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x)$, so $\lim_{x \to -2} f(x)$ exists. ✓ III. $f(-2) = \lim_{x \to -2} f(x) = -5$ ✓ ∴ $f(x)$ is continuous at $x = -2$.

4. Use the three part definition of continuity to graphically justify why $p(x)$ is discontinuous at $x = 0$ and $x = 2$.



$\underline{x=0}$

I. $p(0)$ = hole

$p(0)$ is undefined

$\therefore$ $p(x)$ is discontinuous at $x=0$

$\underline{x=2}$

I. $p(2)=-1$, $p(2)$ is defined ✓

II. $\lim_{x\to 2^-} p(x) = -1$, $\lim_{x\to 2^+} p(x) = 2$

$\lim_{x\to 2} p(x)$ D.N.E.

$\therefore$ $p(x)$ is discontinuous at $x=2$

5. For what values of k and m is the function $g(x)$ everywhere continuous? Use limits to set up your
work.

$$g(x) = \begin{cases} kx^2 + m, & x < -1 \\ e^{\ln(2x+3)}, & -1 \le x \le 3 \\ kx - m, & x > 3 \end{cases}$$

$\underline{x=-1}$

$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^+} g(x)$

$k(-1)^2 + m = e^{\ln(2(-1)+3)}$

$k + m = 1$

$\underline{x=3}$

$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^+} g(x)$

$e^{\ln(2(3)+3)} = k(3) - m$

$9 = 3k - m$

$k + \cancel{m} = 1$

$+\ 3k - \cancel{m} = 9$

$4k = 10$

$\boxed{k = \frac{5}{2}}$

$k + m = 1$

$\frac{5}{2} + m = 1$

$\boxed{m = -3/2}$

Find the value of a that makes each of the functions below everywhere continuous. Write the two limits that must be equal in order for the function to be continuous.

6. $f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \ge -1 \end{cases}$	7. $f(x) = \begin{cases} x^2 + x + a, & x < 2 \\ ax^3 - x^2, & x \ge 2 \end{cases}$
$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$ $4-(-1)^2 = a(-1)^2 - 1$ $4 - 1 = 1a - 1$ $3 = \cancel{a-1}$ $\boxed{a=4}$	$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ $(2)^2 + (2) + a = a(2)^3 - (2)^2$ $4 + 2 + a = 8a - 4$ $6 + a = 8a - 4$ $10 = 7a$ $\boxed{a = 10/7}$