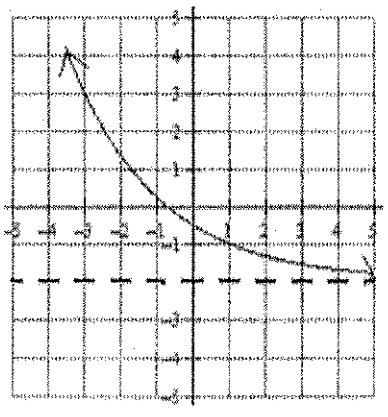
Day 3: Analytical Approach to Finding Limits (continued)

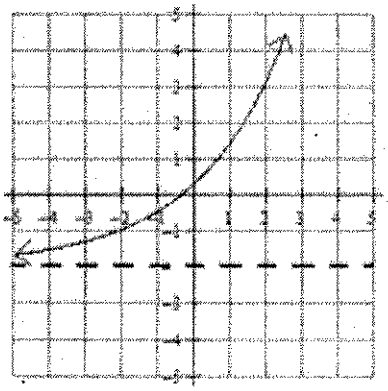
Limits of Exponential Functions:

$f(x) = \left(\frac{2}{3}\right)^{x-1} - 2$



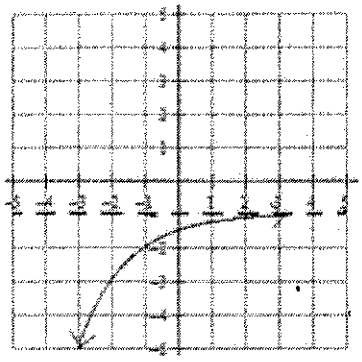
$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -2$

$f(x) = \left(\frac{3}{2}\right)^{x+2} - 2$



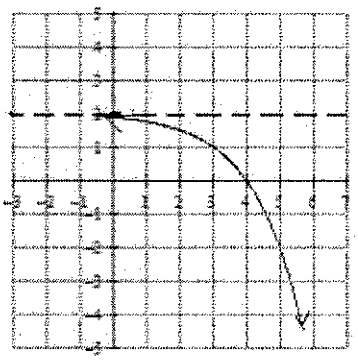
$\lim_{x \to -\infty} f(x) = -2$ $\lim_{x \to \infty} f(x) = \infty$

$f(x) = -\left(\frac{1}{2}\right)^{x+1} - 1$



$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = -1$

$f(x) = -\left(\frac{1}{2}\right)^{-x+3} + 2$



$\lim_{x\to-\infty} f(x) = 2$ $\lim_{x\to\infty} f(x) = -\infty$

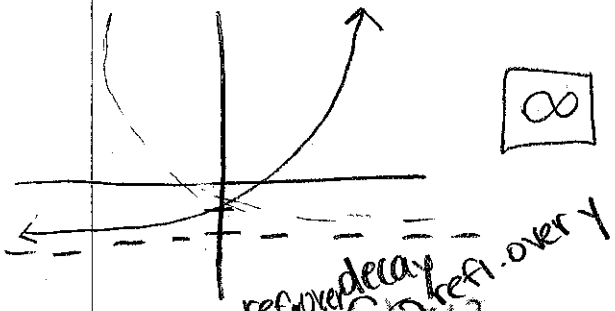
In order to determine a limit as x approaches $-\infty$ or ∞ for an exponential function, you have to determine what the graph will look like. Based on what we have seen above, what are the three possible results of such a limit for an exponential function?

∞	$-\infty$	value of Horiz. Asympt

*refl. over y → refl. over x

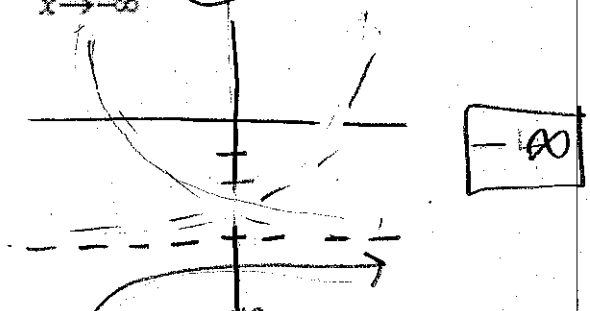
Determine the limit of each of the following exponential functions.

1. $\lim_{x\to\infty} \left(\frac{2}{3}\right)^{x-1} - 2$ — decay, refl. over y, H.A. $y = -2$



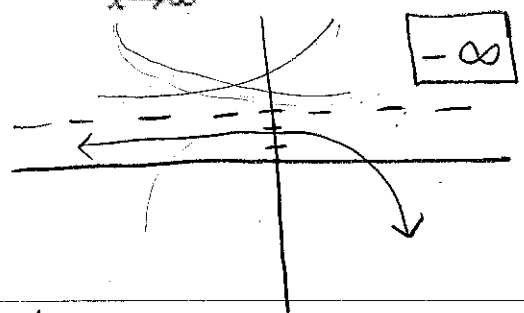
∞

2. $\lim_{x\to-\infty} -(0.4)^{x} - 4$ — refl. over x, decay, H.A at $y = -4$



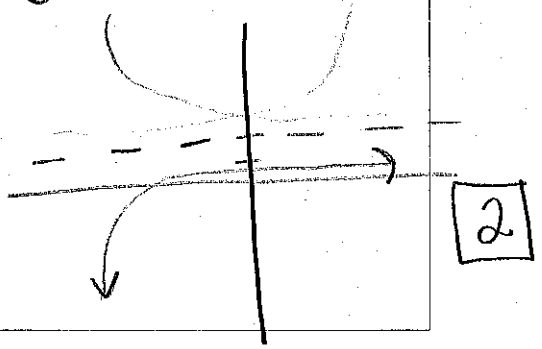
$-\infty$

3. $\lim_{x\to\infty} -\left(\frac{2}{3}\right)^{-x+2} + 3$ — refl. over x, decay, refl. over y



$-\infty$

4. $\lim_{x\to\infty} 2^{x-1} + 2$ — growth



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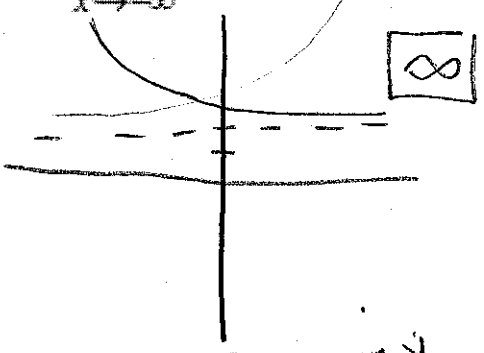
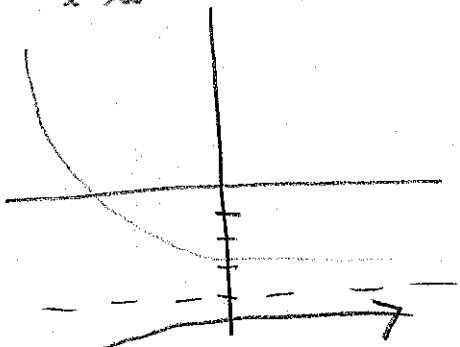
5. $\lim_{x \to -\infty} e^{x-1} + 2$ — growth, refl. over y



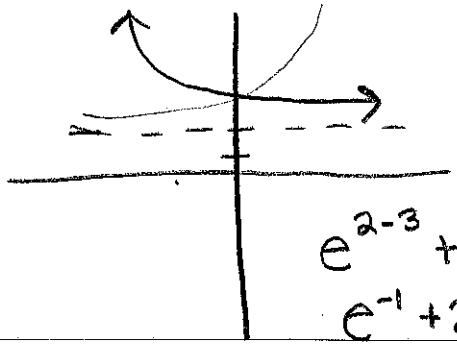
∞

6. $\lim_{x \to \infty} -(0.4)^x - 4$ — refl. over x, decay



-4

7. $\lim_{x \to 3} e^{2-x} + 2$ — growth, refl. over y



$e^{2-3} + 2$

$e^{-1} + 2 = \frac{1}{e} + 2$

8. $\lim_{x \to -2} \left[\left(\frac{1}{2}\right)^{-x-3} + 3\right]$

$\left(\frac{1}{2}\right)^{-2-3} + 3$

$\left(\frac{1}{2}\right)^{-5} + 3$

$(2)^5 + 3 =$

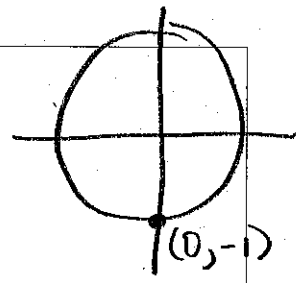
$32 + 3 = 35$

2 2 2 2 2

Limits of Trigonometric Functions:

*Rewrite/Simplify Trig Functions

1. $\lim_{\theta \to \frac{3\pi}{2}} 3\tan\theta\cos\theta = 3\tan\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)$

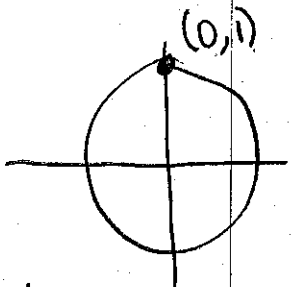


$\left(\frac{-1}{0}\right)$

$3 \cdot \frac{\sin\theta}{\cos\theta} \cdot \cos\theta = 3\sin\theta \longrightarrow \lim_{\theta \to \frac{3\pi}{2}} 3\sin\theta = 3\sin\left(\frac{3\pi}{2}\right)$

$= 3(-1) = -3$

2. $\lim_{\theta \to \frac{\pi}{2}} \frac{\sec\theta\cos\theta}{4\theta} = \frac{\sec\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)}{4\left(\frac{\pi}{2}\right)} = \frac{1}{0}$



$\sec\theta\cos\theta = \frac{1}{\cos\theta} \cdot \cos\theta = 1$

$\lim_{\theta \to \frac{\pi}{2}} \frac{1}{4\theta} = \frac{1}{4\left(\frac{\pi}{2}\right)} = \frac{1}{2\pi}$

3. $\lim_{\theta \to \pi} \dfrac{\cos\theta \tan\theta}{\sin\theta}$

$\cos\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\sin\theta} = 1$

$\lim_{\theta \to \pi} 1 = \boxed{1}$

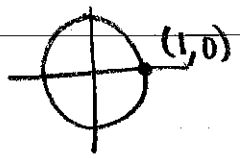
Special Trig Limits

These limits can be used and manipulated to evaluate some limits involving trig.

Memorize them!!

$$\lim_{x \to 0} \frac{\sin cx}{cx} = 1$$

$$\lim_{x \to 0} \frac{\cos cx - 1}{cx} = \lim_{x \to 0} \frac{1 - \cos cx}{cx} = 0$$



1. $\lim_{x\to 0} \frac{e^x \cos x}{4}$

$= \frac{e^0 \cos(0)}{4} = \frac{(1)(1)}{4} = \boxed{\frac{1}{4}}$

2. $\lim_{\theta\to 0} \frac{4\sin 4\theta}{4\theta}$

$\not\exists$

4. $\lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta}$

$4 \cdot 1 = \boxed{4}$

3. $\lim_{x\to 0} \frac{\sin 2x}{3x}$

$= \frac{1}{3} \lim_{X\to 0} 2\frac{\sin 2X}{2X}$

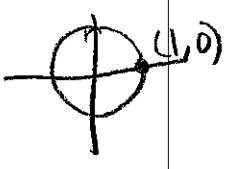
$= \frac{1}{3} \cdot 2 \lim_{X\to 0} \frac{\sin 2X}{2X}$

$= \frac{2}{3}(1) = \boxed{\frac{2}{3}}$

4. $\lim_{\theta\to 0} \frac{2\sin 5\theta}{3\theta} = \frac{2}{3} \lim_{\theta\to 0} 5\frac{\sin 5\theta}{5\theta} = \frac{2}{3} \cdot 5 \lim_{\theta\to 0} \frac{\sin 5\theta}{5\theta}$

$= \frac{2}{3} \cdot 5 \cdot 1 = \boxed{\frac{10}{3}}$

5. $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\theta} = \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta}$

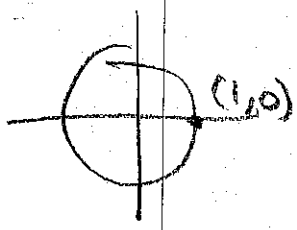


$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} \cdot \lim_{\theta\to 0} \frac{1}{\cos\theta}$

$1 \cdot \frac{1}{\cos 0} = 1 \cdot \frac{1}{1} = \boxed{1}$

$1-\cos^2\theta = \sin^2\theta$

6. $\lim_{\theta\to 0} \frac{2-2\cos^2\theta}{\theta} \rightarrow \frac{2(1-\cos^2\theta)}{\theta} = \frac{2(\sin^2\theta)}{\theta}$



$= 2\sin\theta \cdot \frac{\sin\theta}{\theta}$

$\lim_{\theta\to 0} 2\sin\theta \cdot \lim_{\theta\to 0} \frac{\sin\theta}{\theta}$

$2\sin(0) \cdot 1 = 2(0)\cdot 1 = \boxed{0}$

7. $\lim_{\theta\to 0} \frac{1-\cos\theta+\sin 2\theta}{\theta}$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} + 2\lim_{\theta\to 0} \frac{\sin 2\theta}{2\theta}$

$0 + 2(1) = \boxed{2}$

8. $\lim_{\theta\to 0} \frac{\theta\csc\theta+1}{\theta\csc\theta}$

$\frac{\theta\cdot\frac{1}{\sin\theta}+1}{\theta\cdot\frac{1}{\sin\theta}}$

$\frac{\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta} = \frac{\theta+\sin\theta}{\sin\theta}$

$\frac{\theta}{\sin\theta}$

$\frac{\theta+\sin\theta}{\sin\theta} \cdot \frac{\sin\theta}{\theta} = \frac{\theta+\sin\theta}{\theta}$

$\lim_{\theta\to 0} \frac{\theta}{\theta} + \lim_{\theta\to 0} \frac{\sin\theta}{\theta}$

$1 + 1 = \boxed{2}$

9. $\lim_{x\to 0} \frac{\sin x - \sin x\cos x}{x^2}$

$\frac{\sin x(1-\cos x)}{x^2} \Rightarrow \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1-\cos x}{x}$

$1 \cdot 0 = \boxed{0}$