AP Calculus AB
Unit 1 – Day 3

Name: *Answer Key*

FREE RESPONSE (Calculator NOT Permitted)

Consider the functions below to answer the following questions.

$$F(x) = \frac{3x^2 - 5x - 2}{x^2 - 4} \qquad G(x) = \begin{cases} x^2 + 3|x|, & x < -3 \\ 4x + a, & x > -3 \end{cases}$$

a) Find the value of $\lim_{x\to -3^-} G(x)$. Show you work. (1 point)

1pt: $\lim_{x\to -3^-} G(x) = (-3)^2 + 3|-3| = 9 + 9 = \boxed{18}$

b) Is F(2) = $\lim_{x\to 2} F(x)$? Show you work & explain your reasoning. (2 points)

1pt: $\lim_{x\to 2} \frac{(3x+1)(x-2)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{(3x+1)}{(x+2)} = \frac{3(2)+1}{2+2} = \frac{7}{4}$

$\frac{-6}{-6 \cdot 1}$

$(3x^2 - 6x) + (1x - 2)$

$3x(x-2) + 1(x-2)$

$(3x+1)(x-2)$

1pt: $F(2) = \frac{3(2)^2 - 5(2) - 2}{(2)^2 - 4} = \frac{0}{0}$ since F(2) is undefined and $\lim_{x\to 2} F(x) = \frac{7}{4}$, then $F(2) \neq \lim_{x\to 2} F(x)$.

c) In order for $\lim_{x\to -3} G(x)$ to exist, what two limits must be equal? Find the value(s) of a for which the limit exists. Show you work. (2 points)

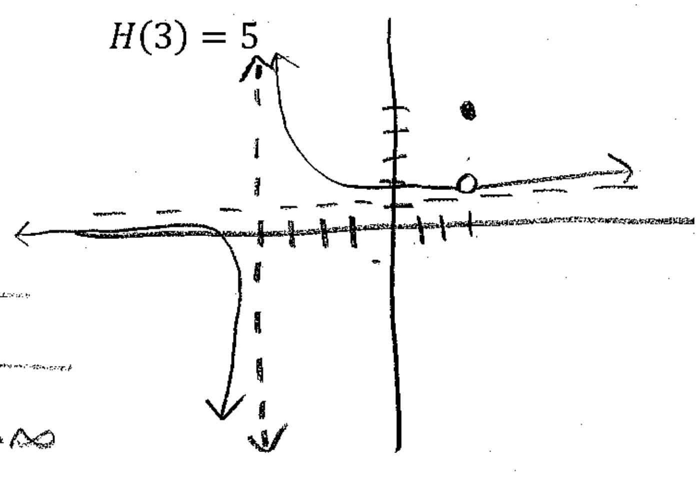
1pt: $\lim_{x\to -3^-} G(x)$ must equal $\lim_{x\to -3^+} G(x)$ in order for $\lim_{x\to -3} F(x)$ to exist.

1pt: $\lim_{x\to -3^-} G(x) = \lim_{x\to -3^+} G(x) \rightarrow 18 = 4(-3) + a \rightarrow \boxed{a = 30}$

d) Draw a graph of a function, H(x) that meets the following criteria. (4 points)

$\lim_{x\to -\infty} H(x) = 1 \qquad \lim_{x\to -4^-} H(x) = -\infty \qquad \lim_{x\to -4^+} H(x) = \infty$

$\lim_{x\to 3} H(x) = 2 \qquad H(3) = 5$



1pt: vertical asymptote at x = −4

1pt: Removable discontinuity at the point (3, 2)

1pt: Point located at (3, 5)

1pt: Graph exhibits horizontally asymptotic behavior as x → −∞