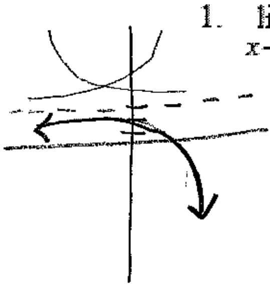
AP Calculus AB Name: Answer Key*

Unit 1 – Day 3 Assignment

Analytical Approach to Finding Limits (continued)

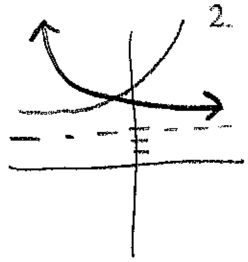
Find the limit of each of the following exponential functions. Sketch a graph of each function to aid in your determination of the limit, if necessary..

1. decay $\lim_{x\to\infty} -(0.5)^{-x-2}+3$



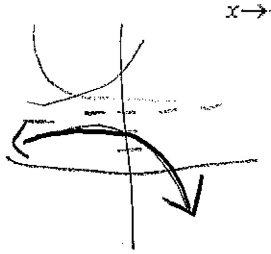
$\boxed{-\infty}$

2. growth $\lim_{x\to\infty} (2)^{-x-2}+3$



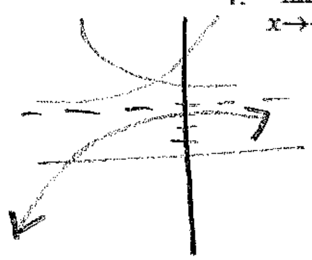
$\boxed{3}$

3. decay $\lim_{x\to-\infty} -\left(\frac{1}{4}\right)^{-x-2}+3$



$\boxed{3}$

4. growth $\lim_{x\to-2} -(3)^{-x-2}+3$



$-(3)^{-(-2)-2}+3$

$= -(3)^0+3$

$= -1+3 = \boxed{2}$

5. $\lim_{x\to-2} \left(\frac{1}{2}\right)^{x+2}-1$

$\left(\frac{1}{2}\right)^{-2+2}-1$

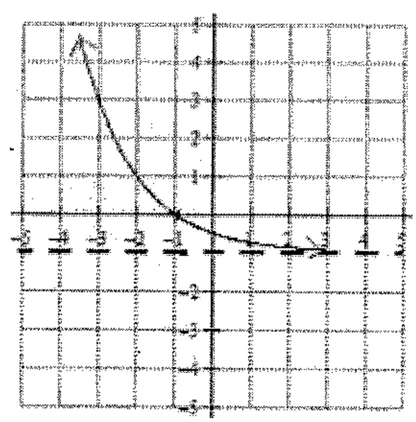
$=\left(\frac{1}{2}\right)^0-1 = 1-1 = \boxed{0}$

6. $\lim_{x\to-1} 2^{-x-2}+2$

$= 2^{-(-1)-2}+2 = 2^{-1}+2$

$= \frac{1}{2}+2 = \boxed{\frac{5}{2}}$

7. Using the graph of $g(x)$ pictured to the right, find each of the following limits.



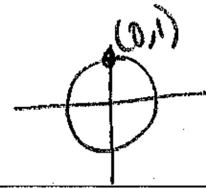
a. $\lim_{x\to\infty} g(x) =$ −1

b. $\lim_{x\to-\infty} g(x) =$ ∞

c. $\lim_{x\to-1} g(x) =$ 0

d. $\lim_{x\to-3} g(x) =$ 3

Find the value of each limit. For a limit that does not exist, state why.



8. $\lim_{x\to 0} \frac{x+\sin x}{x}$

$\lim_{x\to 0} \frac{x}{x} + \lim_{x\to 0} \frac{\sin x}{x}$

$1 + 1$

$\boxed{2}$

9. $\lim_{x\to 3} \begin{cases} 2x^2-3x, & x<3 \\ 8-\cos\left(\frac{\pi x}{3}\right) & x>3 \end{cases}$ $\boxed{9}$

$\lim_{x\to 3^-} f(x) = 2(3)^2 - 3(3) = 9$

$\lim_{x\to 3^+} f(x) = 8 - \cos\left(\frac{3\pi}{3}\right) = 9$

10. $\lim_{\theta\to \frac{\pi}{2}} \frac{\cos^2\theta}{1-\sin\theta} = \frac{(1-\sin^2\theta)}{(1-\sin\theta)}$

$\frac{(1+\sin\theta)\cancel{(1-\sin\theta)}}{\cancel{(1-\sin\theta)}}$ $\lim_{\theta\to\frac{\pi}{2}} (1+\sin\theta)$

$1+\sin\left(\frac{\pi}{2}\right) = 1+1 = \boxed{2}$

11. $\lim_{\theta\to 0} \frac{2\sin 3\theta}{\theta}$

$3 \cdot 2 \lim_{\theta\to 0} \frac{\sin 3\theta}{3\theta}$

$6(1) = \boxed{6}$

12. $\lim_{x\to 0} \frac{\sin x}{2x^2-x}$

$\frac{\sin x}{x(2x-1)}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{2x-1}$

$1 \cdot \frac{1}{2(0)-1} = \boxed{-1}$

13. $\lim_{x\to 0} \frac{5x+\sin 3x}{x}$

$\lim_{x\to 0} \frac{5x}{x} + 3\lim_{x\to 0} \frac{\sin 3x}{3x}$

$5+3(1) = \boxed{8}$

14. $\lim_{x\to 0} \frac{2\sin 4x}{3x}$

$4 \cdot \frac{2}{3} \lim_{x\to 0} \frac{\sin 4x}{4x}$

$4\left(\frac{2}{3}\right)(1) = \boxed{\frac{8}{3}}$

15. $\lim_{x\to 0} \frac{\sin 2x}{6x}$

$2 \cdot \frac{1}{6} \lim_{x\to 0} \frac{\sin 2x}{2x}$

$2\left(\frac{1}{6}\right)(1) = \frac{2}{6} = \boxed{1/3}$

16. $\lim_{\theta\to 0} \frac{\cos\theta\tan\theta}{3\theta}$

$\cancel{\cos\theta} \cdot \frac{\sin\theta}{\cancel{\cos\theta}}$ $\lim_{\theta\to 0} \frac{\sin\theta}{3\theta}$

$\frac{1}{3} \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \frac{1}{3}(1) = \boxed{1/3}$

17. $\lim_{\theta\to 0} \frac{3-3\cos\theta}{\theta}$

$\frac{3(1-\cos\theta)}{\theta}$

$3 \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}$

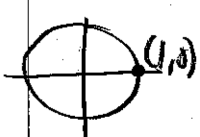
$3(0) = \boxed{0}$

18. $\lim_{\theta\to\frac{\pi}{2}} \frac{\cos\theta}{\cot\theta}$ $\frac{\cos\theta}{\sin\theta}$

$\cos\theta \cdot \frac{\sin\theta}{\cancel{\cos\theta}}$

$\lim_{\theta\to\frac{\pi}{2}} \sin\theta = \sin\left(\frac{\pi}{2}\right) = \boxed{1}$

19. $\lim_{\theta\to 0} \frac{1-\tan\theta}{\sin\theta-\cos\theta}$



$\frac{1-\tan(0)}{\sin(0)-\cos(0)} = \frac{1-0}{0-1}$

$= \frac{1}{-1} = \boxed{-1}$

20. $\lim_{c\to 3} \frac{c^3-27}{c-3}$

$\frac{\cancel{(c-3)}(c^2+3c+9)}{\cancel{(c-3)}}$

$\lim_{c\to 3} (c^2+3c+9)$

$(3)^2+3(3)+9$

$9+9+9 = \boxed{27}$

21. $\lim_{x\to -1} \frac{(x+3)^3-8}{x+1}$

$\frac{[\cancel{(x+3)-2}][(x+3)^2+2(x+3)+4]}{\cancel{(x+1)}}$

$\lim_{x\to -1} (x+3)^2+2(x+3)+4 = (-1+3)^2+2(-1+3)+4$

$= 4+4+4 = \boxed{12}$