AP Calculus AB
Unit 1 – Day 2 Assignment

Name: Answer Key*

Analytical Approach to Finding Limits

Find the value of each limit. For a limit that does not exist, state why.

1. $\lim\limits_{x\to -\frac{1}{2}} 3x^2(2x-1)$

$3(-1/2)^2\left(2\left(-\frac{1}{2}\right)-1\right)$

$3\left(\frac{1}{4}\right)(-2)$

$\frac{3}{4}(-2) = \boxed{-\frac{3}{2}}$

2. $\lim\limits_{x\to -1} x^3+2x^2-3x+3$

$(-1)^3+2(-1)^2-3(-1)+3$

$-1+2(1)+3+3$

$=\boxed{7}$

3. $\lim\limits_{x\to -2} (x-6)^{2/3}$

$(-2-6)^{2/3} = (-8)^{2/3}$

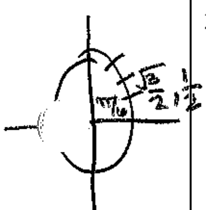
$\sqrt[3]{(-8)^2} = \sqrt[3]{64}$

$=\boxed{4}$

4. $\lim\limits_{x\to 2} \frac{x^2+5x+6}{x+2} = \frac{(2)^2+5(2)+6}{2+2}$

$= \frac{20}{4} = \boxed{5}$

5. $\lim\limits_{\theta\to \frac{\pi}{6}} \theta^2\tan\theta$



$\left(\frac{\pi}{6}\right)^2\cdot\tan\left(\frac{\pi}{6}\right)$

$\downarrow$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}$

$\frac{\pi^2}{36}\cdot\frac{1}{\sqrt{3}} = \boxed{\frac{\pi^2}{36\sqrt{3}}}$

6. $\lim\limits_{x\to 0} \frac{(x+4)^2-16}{x} = \frac{x^2+8x+16-16}{x} = \frac{x^2+8x}{x}$

$= \frac{\cancel{x}(x+8)}{\cancel{x}}$

$\lim\limits_{x\to 0}(x+8) = 0+8 = \boxed{8}$

7. $\lim\limits_{x\to 1} \frac{x-1}{x^2-1} = \frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

$\lim\limits_{x\to 1}\frac{1}{(x+1)} = \frac{1}{1+1} = \boxed{1/2}$

8. $\lim\limits_{x\to 2} \frac{x^2-3x+2}{x^2-4} = \frac{\cancel{(x-2)}(x-1)}{(x+2)\cancel{(x-2)}}$

$\lim\limits_{x\to 2}\frac{(x-1)}{(x+2)} = \frac{2-1}{2+2} = \boxed{\frac{1}{4}}$

9. $\lim_{x\to 0} \frac{5x^3+8x^2}{3x^4-16x^2} = \frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$

$\lim_{x\to 0} \frac{(5x+8)}{(3x^2-16)} = \frac{5(0)+8}{3(0)^2-16} = \frac{8}{-16}$

$= \boxed{-1/2}$

10. $\lim_{x\to 0} \frac{\frac{1}{x+2}-\frac{1}{2}}{x}$

$\frac{1}{x+2}\frac{(2)}{(2)} + \frac{-1}{2}\frac{(x+2)}{(x+2)}$

$\frac{2}{2(x+2)} + \frac{-x-2}{2(x+2)} = \frac{\frac{-x}{2(x+2)}}{x}$

$\frac{-\cancel{x}}{2(x+2)} \cdot \frac{1}{\cancel{x}} = \frac{-1}{2(x+2)}$

$\lim_{x\to 0} -\frac{1}{2(x+2)} = \frac{-1}{2(0+2)} = \boxed{\frac{-1}{4}}$

11. $\lim_{x\to 0} \frac{(2+x)^3-8}{x}$

$(2+x)(2+x)$

$(4+4x+x^2)(2+x)$

$8+4x+8x+4x^2+2x^2+x^3$

$x^3+6x^2+12x+8-8$

$\frac{x^3+6x^2+12x}{x} = \frac{\cancel{x}(x^2+6x+12)}{\cancel{x}}$

$\lim_{x\to 0}(x^2+6x+12) = 0^2+6(0)+12$

$= \boxed{12}$

12. $\lim_{h\to 0} \frac{(x+h)^2+2(x+h)-3-(x^2+2x-3)}{h}$

$\cancel{x^2}+2xh+h^2+\cancel{2x}+2h\cancel{-3}\cancel{-x^2}\cancel{-2x}\cancel{+3}$

$\frac{2xh+h^2+2h}{h} = \frac{\cancel{h}(2x+h+2)}{\cancel{h}}$

$\lim_{h\to 0}(2x+h+2) = \boxed{2x+2}$

13. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if $f(x)=3x^2-2x$

$f(x+h) = 3(x+h)^2-2(x+h)$

$3(x^2+2xh+h^2)-2x-2h$

$3x^2+6xh+3h^2-2x-2h$

$\cancel{3x^2}+6xh+3h^2\cancel{-2x}-2h\cancel{-3x^2}\cancel{+2x}$

$\frac{6xh+3h^2-2h}{h}$

$\frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$

$\lim_{h\to 0}(6x+3h-2) = \boxed{6x-2}$

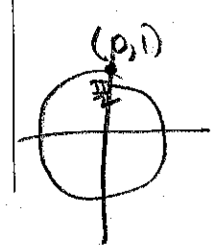
14. $\lim_{x\to 2} f(x)$ if $f(x) = \begin{cases} 2x^2-4x, & x<2 \\ 4\sin\left(\frac{\pi x}{4}\right) & x>2 \end{cases}$

$\lim_{x\to 2^-} = 2(2)^2-4(2) = 8-8 = 0$

$\lim_{x\to 2^+} = 4\sin\left(\frac{\pi(2)}{4}\right) = 4\sin\left(\frac{\pi}{2}\right)$

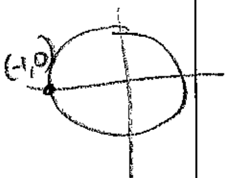
$= 4(1)$

$= 4$



$\boxed{\lim_{x\to 2} f(x) \text{ D.N.E.}}$

15. $\lim_{x\to 3} e^{x}\cos\left(\frac{\pi x}{3}\right)$



$e^{3}\cdot\cos\left(\frac{3\pi}{3}\right)$

$= e^{3}\cos(\pi)$

$= e^{3}(-1)$

$= \boxed{-e^{3}}$

16. $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} \cdot \frac{(\sqrt{x+3}+2)}{\sqrt{x+3}+2} = \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$

$= \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x+3}+2)}$

$= \frac{1}{\sqrt{x+3}+2}$

$\lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{\sqrt{1+3}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$

17. $\lim_{x\to 3^{+}} \frac{x+3}{x-3}$

can't factor

V.A.

plug in 3.1 $\frac{3.1+3}{3.1-3} = \frac{+}{+} =$ pos

$\boxed{\infty}$

18. $\lim_{x\to -3^{+}} \frac{2x^{2}-9x+9}{x^{2}-9} = \frac{(2x-3)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$

$\lim_{x\to -3^{+}} \frac{(2x-3)}{(x+3)}$

V.A.

plug in -2.9

$\frac{2(-2.9)-3}{-2.9+3} = \frac{-}{+} =$ neg

$\boxed{-\infty}$

$\begin{array}{c|c} \multicolumn{2}{c}{18} \\ -6 & -3 \end{array}$

$(2x^{2}-6x)(3x+9)$

$2x(x-3)-3(x-3)$

$(2x-3)(x-3)$

19. $\lim_{x\to 0} \frac{\frac{1}{x-2}+\frac{1}{2}}{x}$

$\frac{1}{x-2}\frac{(2)}{(2)} + \frac{1}{2}\frac{(x-2)}{(x-2)}$

$\frac{2+x-2}{2(x-2)} = \frac{\cancel{x}}{2(x-2)} \cdot \frac{1}{\cancel{x}}$

$\lim_{x\to 0} \frac{1}{2(x-2)} = \frac{1}{2(0-2)} = \boxed{\frac{1}{-4}}$

20. $\lim_{x\to -2} \begin{cases} 2-x, & x<-2 \\ x^{2}-2x, & x>-2 \end{cases}$

$\lim_{x\to -2^{-}} \quad 2-(-2) = 4$

$\neq$

$\lim_{x\to -2^{+}} \quad (-2)^{2}-2(-2) = 4+4 = 8$

$\boxed{\text{D.N.E}}$

21. If $\lim_{x\to 3} f(x) = 2$ and $\lim_{x\to 3} g(x) = -4$, find each of the following limits. Show your analysis applying the properties of limits.

a. $\lim_{x\to 3}\left[\frac{5f(x)}{g(x)}\right]$ $\frac{5(2)}{-4} = \frac{10}{-4} = \boxed{\frac{5}{-2}}$	b. $\lim_{x\to 3}[f(x)+2g(x)]$ $2+2(-4)$ $2+-8 = \boxed{-6}$	c. $\lim_{x\to 3}\sqrt{4f(x)}$ $\sqrt{4(2)} = \sqrt{8}$ $\boxed{2\sqrt{2}}$
d. $\lim_{x\to 3}\frac{g(x)}{8}$ $\frac{-4}{8} = \boxed{-1/2}$	e. $\lim_{x\to 3}[3f(x)-g(x)]$ $3(2)-(-4)$ $6+4$ $\boxed{10}$	f. $\lim_{x\to 3}\left[\frac{f(x)g(x)}{12}\right]$ $\frac{(2)(-4)}{12} = \frac{-8}{12}$ $= \frac{-4}{6} = \boxed{\frac{-2}{3}}$

22. If $\lim_{x\to 4} f(x) = 0$ and $\lim_{x\to 4} g(x) = 3$, find each of the following limits. Show your analysis applying the properties of limits.

a. $\lim_{x\to 4}\left[\frac{g(x)}{f(x)-1}\right]$ $\frac{3}{0-1} = \frac{3}{-1} = \boxed{-3}$	b. $\lim_{x\to 4} xf(x)$ $4(0) = \boxed{0}$
c. $\lim_{x\to 4}[g(x)+3]$ $3+3 = \boxed{6}$	d. $\lim_{x\to 4} g^2(x)$ $(3)^2 = \boxed{9}$