AP Calculus BC Name: Answer Key*

Unit 11 – REVIEW WARM-UP

velocity = $\langle x'(t), y'(t) \rangle$

$x'(t) = \cos(t^2)$

$y'(t) = e^{0.5t}$

1. Calculator Active

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

$x(1) = 3 \quad y(1) = 5$

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(a) $x(2) = ?$

$x(2) - x(1) = \int_1^2 \cos(t^2)\,dt$ (math 9)

$x(2) - 3 = -0.443$

$\boxed{x(2) = 2.557}$

(b) slope = 2

$\frac{dy}{dx} = 2 \rightarrow \frac{dy}{dx} = \frac{e^{0.5t}}{\cos(t^2)}$ $= 2$

y_1 y_2

intersection

$\boxed{t = 0.840}$

(c) speed = 3, $t = ?$

$3 = \sqrt{(\cos(t^2))^2 + (e^{0.5t})^2}$

y_1 y_2

intersection

$\boxed{t = 2.194}$

(d) Total Distance

$\int_0^1 \sqrt{(\cos(t^2))^2 + (e^{.5t})^2}\,dt$

(math 9)

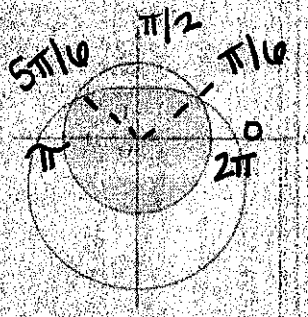
$\boxed{1.595}$

$r=3$

$r=4-2\sin\theta$

2. Calculator Active

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(a) TOP area: $2\left[\frac{1}{2}\int_0^{\pi/6}(3)^2\,d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2}(4-2\sin\theta)^2\,d\theta\right]$

$2(2.356 + 2.930) = 10.572$

Bottom Area: $\int_{\pi}^{2\pi}(3)^2\,d\theta = 14.137$

Area $= 10.572 + 14.137 = \boxed{24.709}$

(b) $x = r\cos\theta$

$x = (4-2\sin\theta)(\cos\theta)$, $\theta = t^2$

$(4-2\sin(t^2))(\cos(t^2)) = -1$

$y_1 \qquad y_2$

Intersection

$\boxed{t = 1.428}$ $\quad 1 \le t \le 2$

$\theta = t^2$

(c) position vector $\langle x(t), y(t)\rangle$

$x = r\cos\theta$ $\qquad y = r\sin\theta$

$x = (4-2\sin\theta)(\cos\theta)$ $\qquad y = (4-2\sin\theta)(\sin\theta)$

$x = (4-2\sin t^2)(\cos t^2)$ $\qquad y = (4-2\sin t^2)(\sin t^2)$

$\boxed{\langle (4-2\sin t^2)(\cos t^2),\ (4-2\sin t^2)(\sin t^2)\rangle}$

velocity vector $\langle x'(t), y'(t)\rangle$ at $t = 1.5$:

$\langle x'(1.5), y'(1.5)\rangle$

math 8

$\boxed{\langle -8.072, -1.673\rangle}$